Graph Shift Operator for Power System Applications

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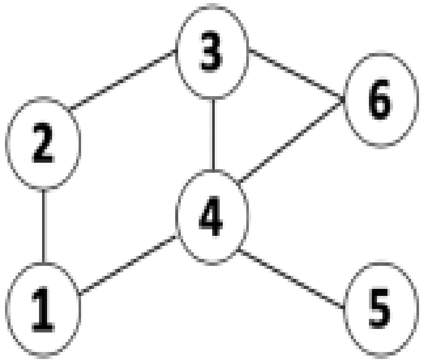
TU Delft IEPG Intelligent Electrical Power Grids

What is a graph shift operator?

A matrix $\boldsymbol{S}$ which represents the structure of a graph

A matrix $S \in \mathbb{R}^{n \times n}$ is called a *Graph Shift Operator* (GSO) if it satisfies:
$S_{ij} = 0$ for $i \neq j$ and $(i, j) \notin E$.



$$\mathbf{W} = \begin{bmatrix} W_{11} & W_{12} & 0 & W_{14} & 0 & 0 \\ W_{21} & W_{22} & W_{23} & 0 & 0 & 0 \\ 0 & W_{32} & W_{33} & W_{34} & 0 & W_{36} \\ W_{41} & 0 & W_{43} & W_{44} & W_{45} & W_{46} \\ 0 & 0 & 0 & W_{54} & W_{55} & 0 \\ 0 & 0 & W_{63} & W_{64} & 0 & W_{66} \end{bmatrix}$$

GSOs for GNN

Adjacency matrix : A

Laplacian matrix : $L = D - A$

Normalized Adjacency matrix : $D^{-1/2} A D^{-1/2}$ *

Symmetric normalized Laplacian matrix : $I_n - D^{-1/2} A D^{-1/2}$

Random walk normalized Laplacian : $I_n - D^{-1} A$

* Semi supervised classification with graph convolution networks, Kipf and Welling, 2016

Basics of GNN

General graph convolutional layer

$$X^{l+1} = \sigma\left[\sum_{k=0}^{\infty} S^k X^l W_k^L\right]$$

where:

k : number of hops

l : layer number

X : graph feature

S : GSO

W : learnable model weight

Linear graph filter :

$$Z = \sum_{k=0}^{\infty} h_k S^k X$$

Message Passing Graph Neural Network

Three Steps:

- Message Construction
- Message Aggregation
- Message Update

$$x_i' = \gamma\left(x_i \oplus_{j \in N(i)} \phi(x_i, x_j, e_{ji})\right)$$

where:

ϕ : message construction function (e.g., MLP)

$\oplus$: message aggregation function (e.g., sum)

γ : message update function (e.g., MLP)

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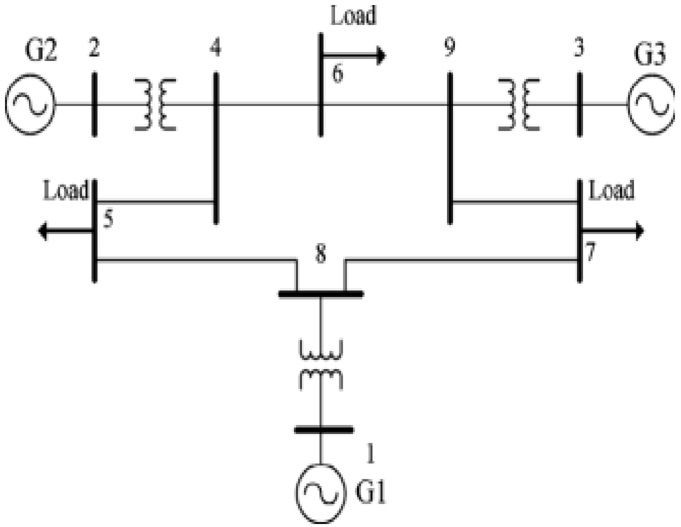
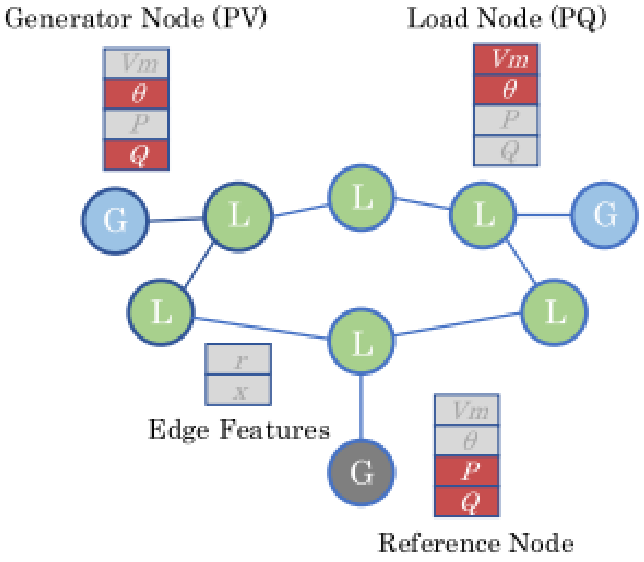
GNN for Power System Analysis

IEEE 9 bus network



Graph equivalent *



* PoweflowNet: Powerflow approximation using message passing GNN, N.Lin et al, 2024

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Best GSO?

“No one matrix is the best because each matrix has its limitations in that there is some property which the matrix cannot always determine” *

- We should choose a matrix that best fits the properties we need
- We are interested in stability of GNNs to topological variations in power systems

* Spectral Graph Theory, Butler and Chung, 2013

GNN Stability

- We are interested in perturbation of the power network graph due to random loss of an edge (N-1 contingency)

- If there is a perturbation of the S matrix, how stable is the graph filter?

Lipschitz filters

Standard Lipschitz Filter:

$$|h(\lambda_2) - h(\lambda_1)| \leq C|\lambda_2 - \lambda_1|$$

- Frequency response is at most linear

Integral Lipschitz Filter:

$$|\lambda h'(\lambda)| \leq C$$

- Frequency response at large λ is flat
- Stable to stochastic graph perturbations

* Stability properties of Graph Neural Networks, F.Gama et al, 2020

Theoretical Insight

Expected difference between GCN output on a graph S and its subgraph $\tilde{S}$ is:

$$\mathbb{E}\left[\|\Phi(x, S, H) - \tilde{\Phi}(x, \tilde{S}, H)\|_2^2\right] \leq C(1-p)$$

$$\|x\|_2^2 + O((1-p)^2)$$

where:

- C is a constant dependent on GSO choice

* Stability of Graph Convolution Neural Networks to Stochastic Perturbation, Z.Gao et al, 2021

Theoretical Insight

- Filter *H* is learnt (W) in the case of GNN, not explicitly chosen
- *C* can be made smaller by learning a more stable filter
- We can use power system physics to bias learning

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GSO for power system

Consider two GSOs with the following frequency spectrum:

- S_1: Linear eigenvalue decay

$$\lambda_i \approx -\beta i + c$$

- S_2: Exponential eigenvalue decay

$$\lambda_i \approx e^{-\alpha i}$$

GSO 1 – Linear Decay

Properties of GSO with $\lambda_i \approx -\beta \boldsymbol{i} + \mathbf{c}$:

- Rate of change between consecutive eigenvalues:

$$|\lambda_i - \lambda_{i+1}| \approx \beta$$

- Harder to satisfy integral Lipschitz condition

GSO 2 – Exponential Decay

Properties of GSO with $\lambda_i \approx e^{-\alpha i}$:

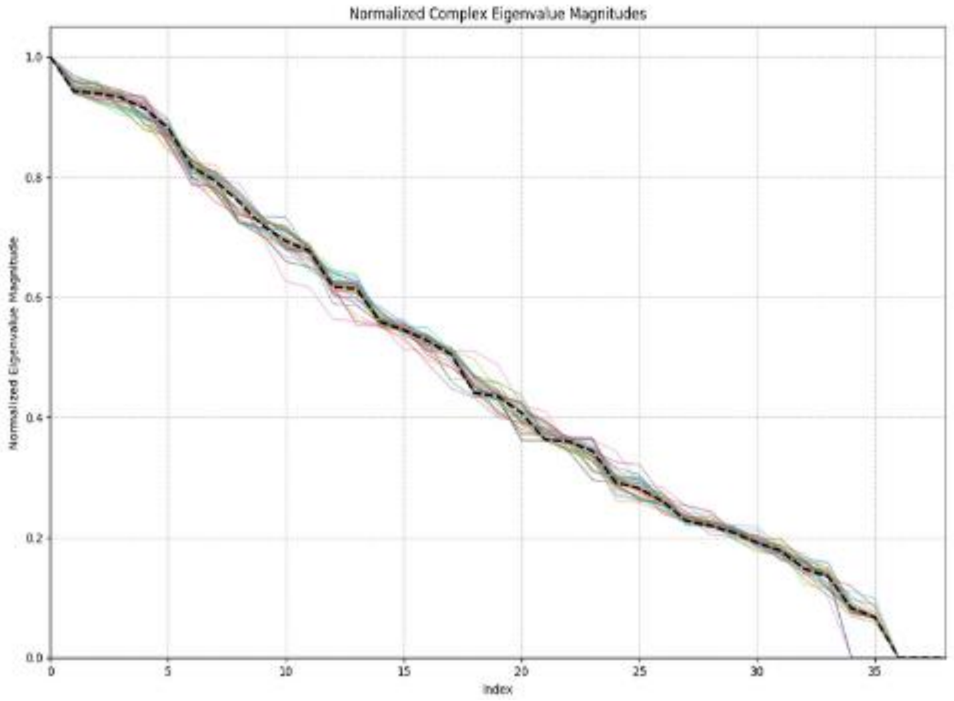
- Rate of change between consecutive eigenvalues:

$$|\lambda_i - \lambda_{i+1}| \approx |e^{-\alpha i}(1 - e^{-\alpha})|$$

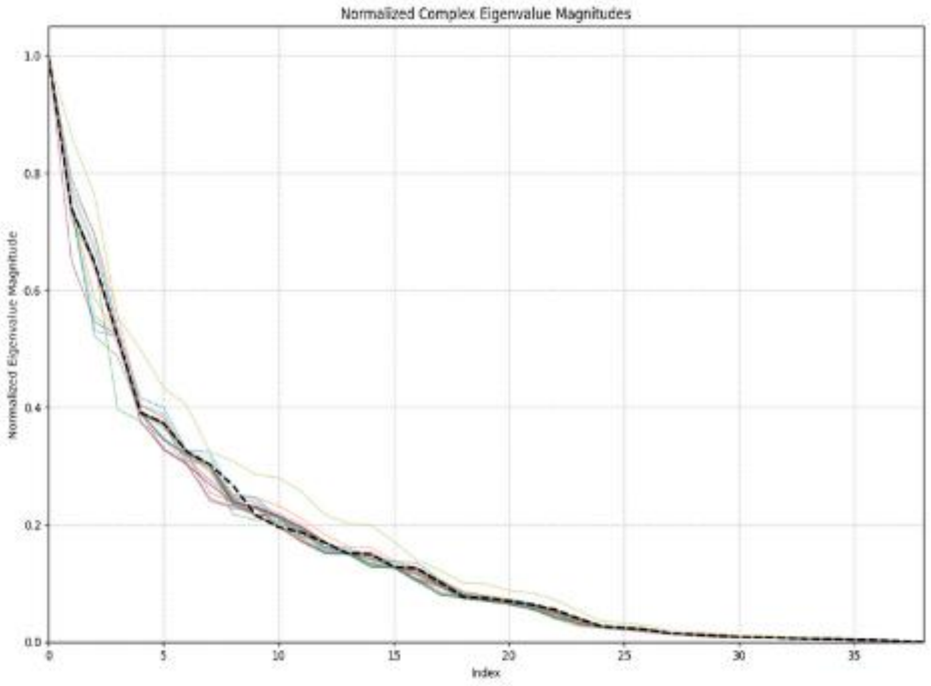
- Rapid decay of high frequencies
- May more naturally satisfy integral Lipschitz condition with smaller C

Comparing the spectrum of different GSOs (IEEE 39)

$D^{-1\backslash 2} A D^{-1\backslash 2}$



B → Power network susceptance matrix



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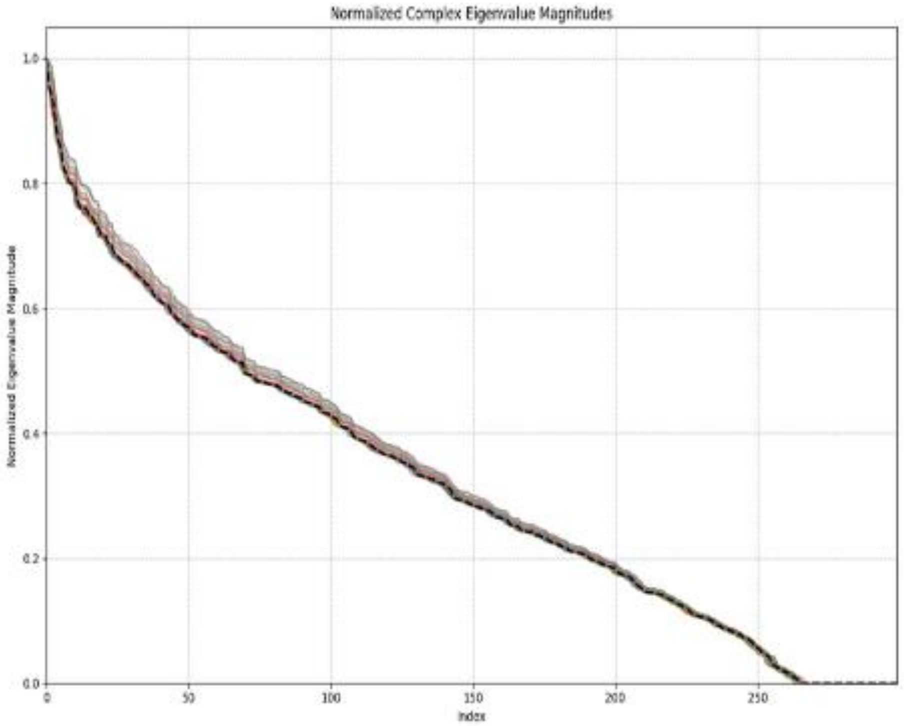
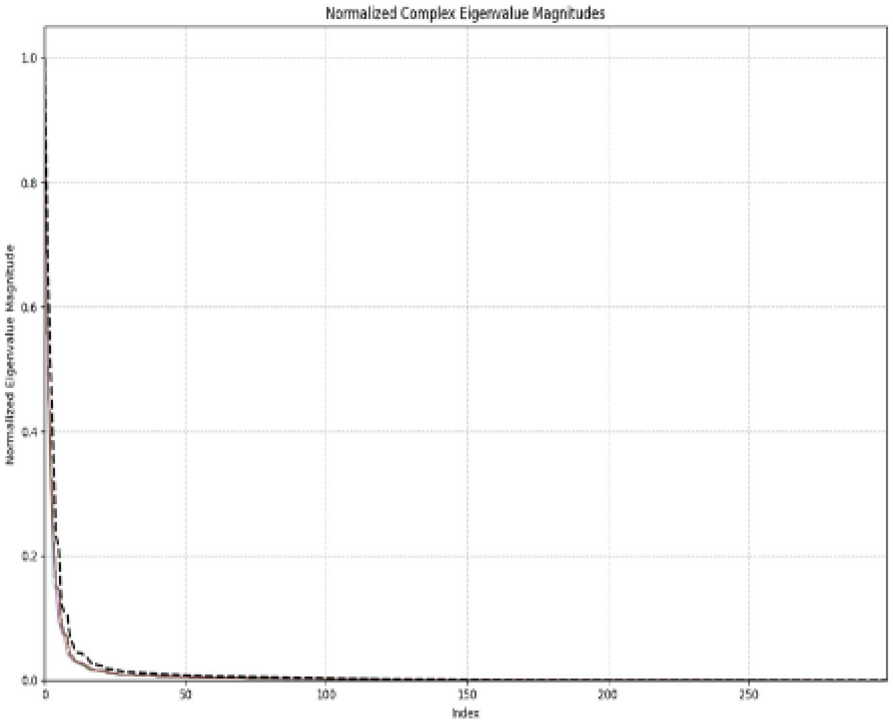
Comparing the spectrum of different GSOs (IEEE 300)

$D^{-1\backslash 2} A D^{-1\backslash 2}$



B



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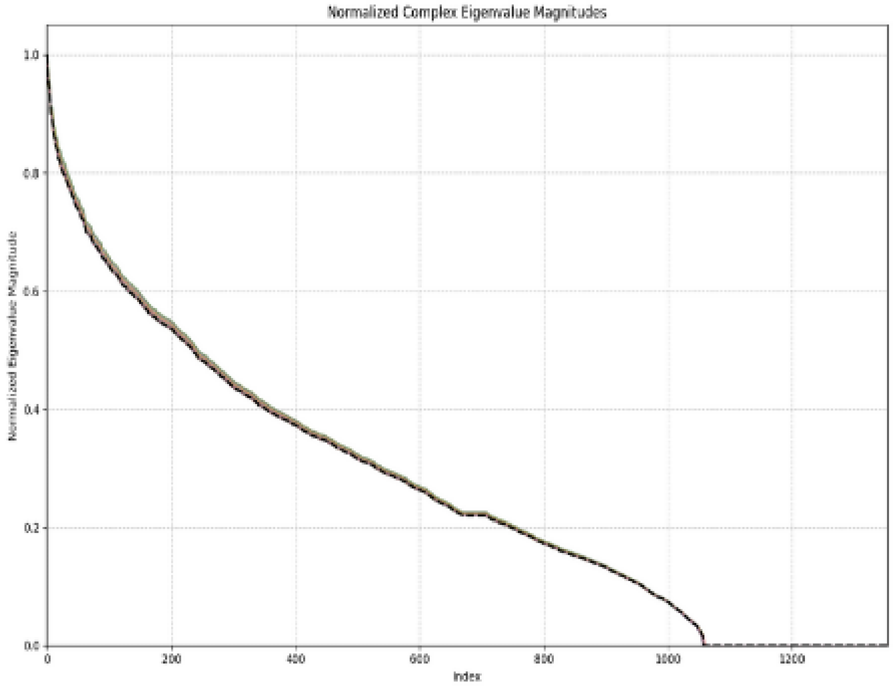
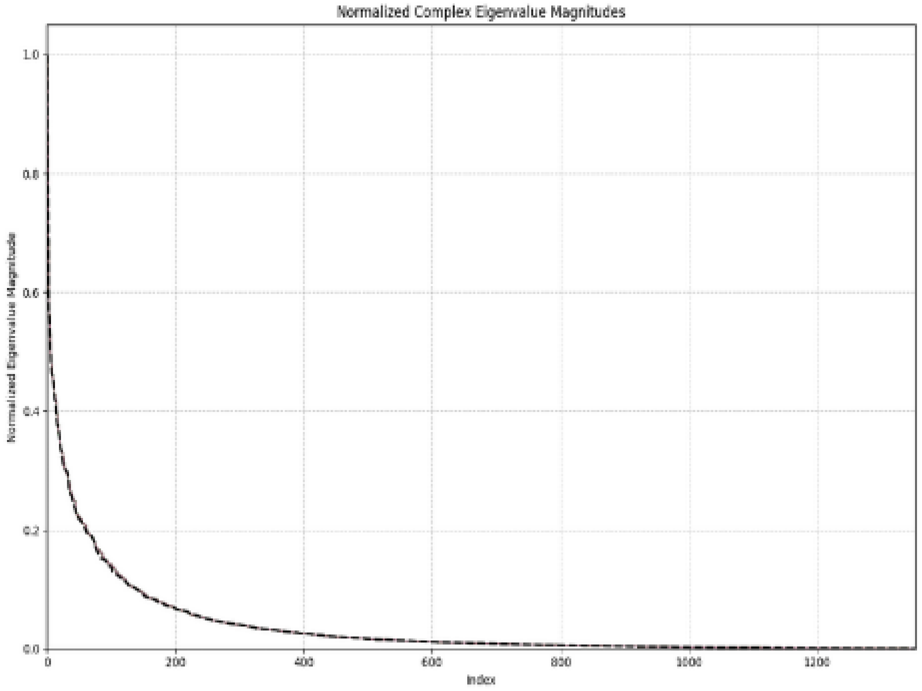
Comparing the spectrum of different GSOs (IEEE 1354)

$D^{-1\backslash 2} A D^{-1\backslash 2}$



B



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Case study

Research Question: Is the physics based GSO more stable to N-1 line contingencies when predicting ACOPF solutions?

ACOPF

- Predict generator power injections (real and reactive)
- Predict voltages at the bus (magnitude and angle)
- Voltage output is topology dependent and exhibit strong locality property *

* Topology aware GNN for learning feasible and adaptive ACOPF Solutions, S.Liu et al, 2023

Dependence of voltage on B

From fast decoupled power flow equations:

$$\Delta\theta = [B']^{-1}\Delta P$$
$$\Delta V = [B'']^{-1}\Delta Q$$

where:

- B' and B'' are modified susceptance matrices
- Hence we can consider B as the underlying GSO

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Case study

Large scale ACOPF Dataset recently released by Deepmind *

We consider the medium sized IEEE 118 bus grid

- Vanilla fully connected neural network
- Message passing GNN with normalized adjacency matrix
- Message passing GNN with B-matrix as the GSO

* OPFData: Large scale dataset for ACOPF solutions with topological perturbation, S.Lovett et al, 2024

Results

All results in three settings (A,B,C)

Scalability of FNN to large grids challenging (1.2M parameters)

Train – Test setting	MSE
Full – Full topology	4.0e-3 - 3.7e-3
N-1 – N-1 topology	2.0e-3 - 2.4e-3
Full – N-1 topology	4.0e-3 - 6.4e-3

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Results

- GNN requires only a fraction of the model complexity to achieve similar accuracy (86K parameters)
- More scalable to large grids
- GNN with normalized A matrix as GSO

Train – Test setting	**MSE**
Full – Full topology	3.0e-3 - 2.6e-3
N-1 – N-1 topology	4.0e-3 - 4.0e-3
Full – N-1 topology	3.0e-3 - 5.6e-3

Results

- GNN with B matrix as GSO
- Number of parameters is independent of the graph
- Combination of Message passing with Graph convolution

Train – Test setting	MSE
Full – Full topology	3.0e-3 - 2.6e-3
N-1 – N-1 topology	4.0e-3 - 4.0e-3
Full – N-1 topology	3.0e-3 - 3.4e-3

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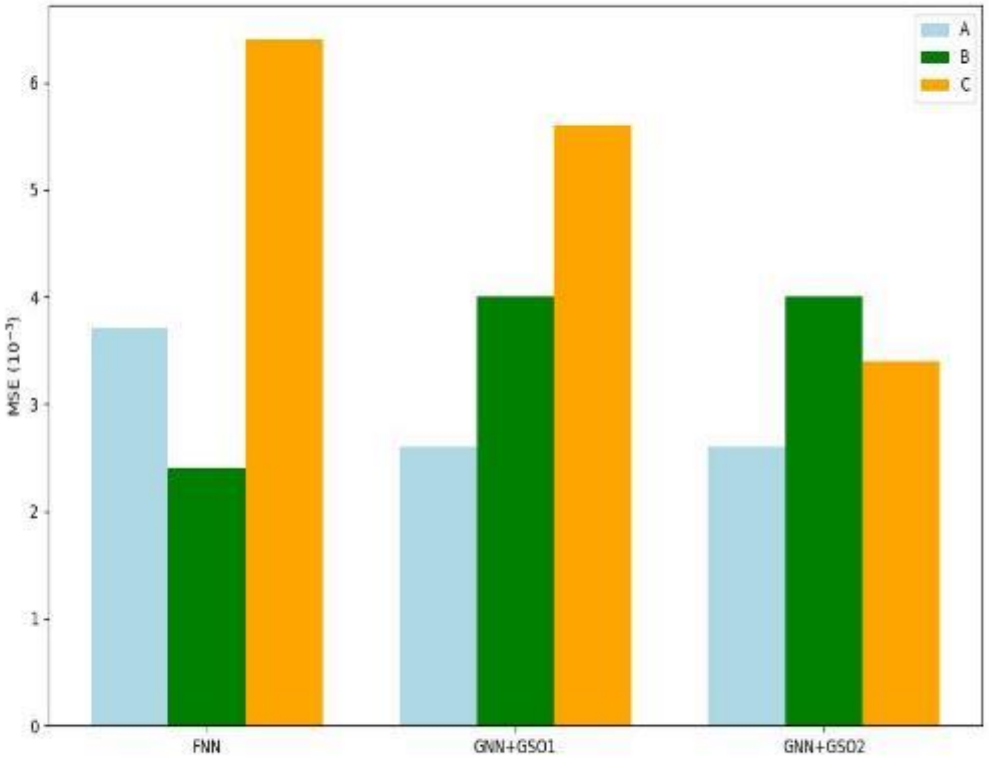
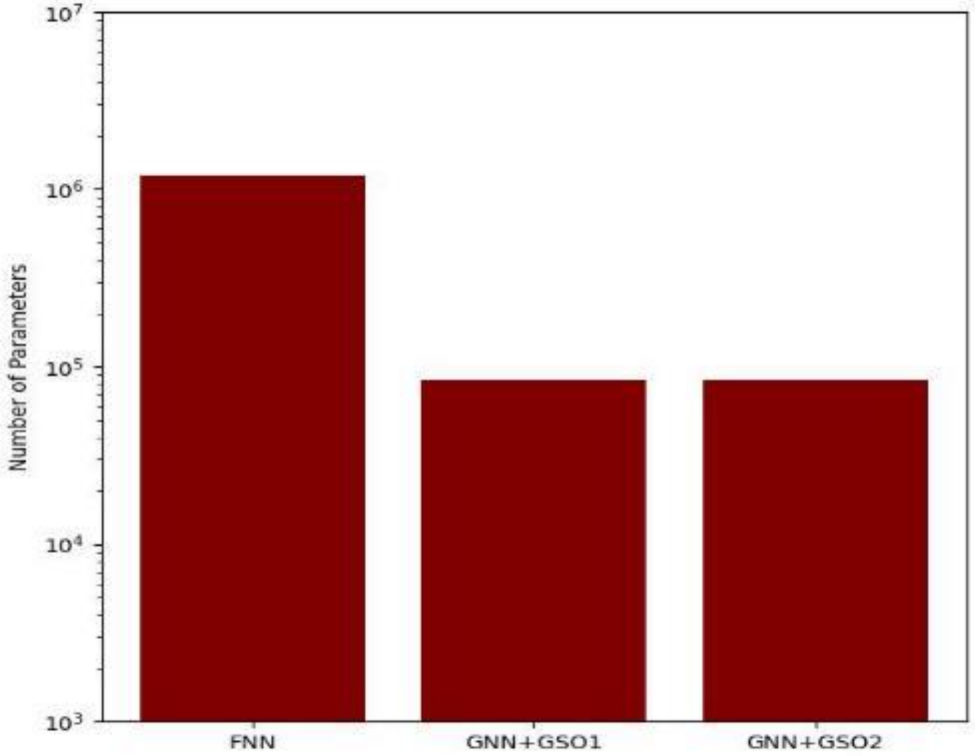
Results

Model Accuracy



Model Parameters



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Conclusions

- GNNs may be used to learn fast approximate ACOPF solutions
- The choice of GSO may affect stability of GNNs to perturbations
- The use of physics informed GSO may improve stability of GNN to topological perturbations

Thank you for your attention

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