Diffusion Backbones of Temporal Higher-order Networks

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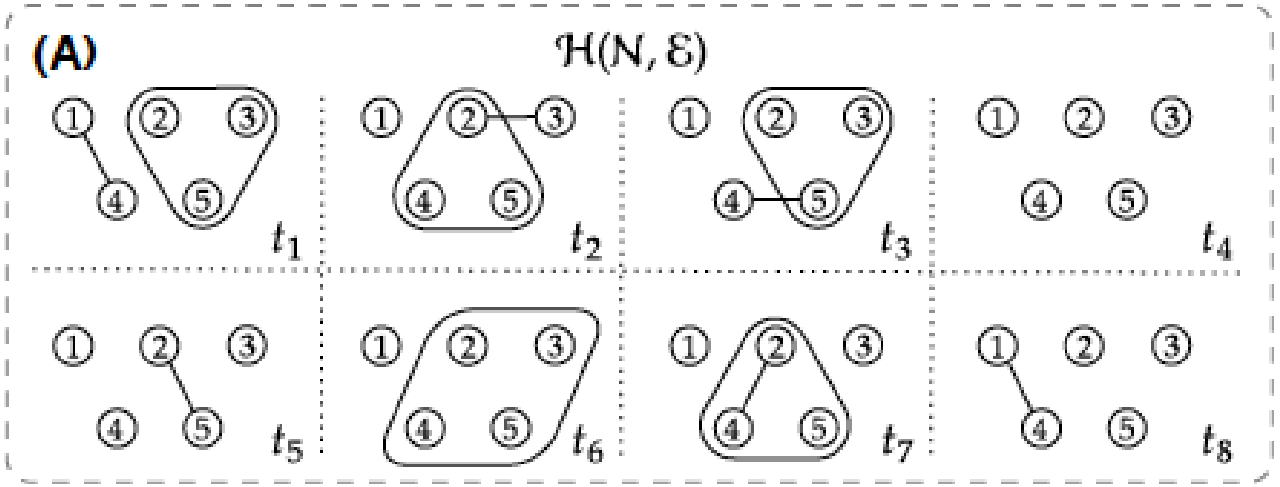
Temporal higher-order network (THN)
(Temporal hypergraph)

THN
$\mathcal{H}(N, \mathcal{E})$



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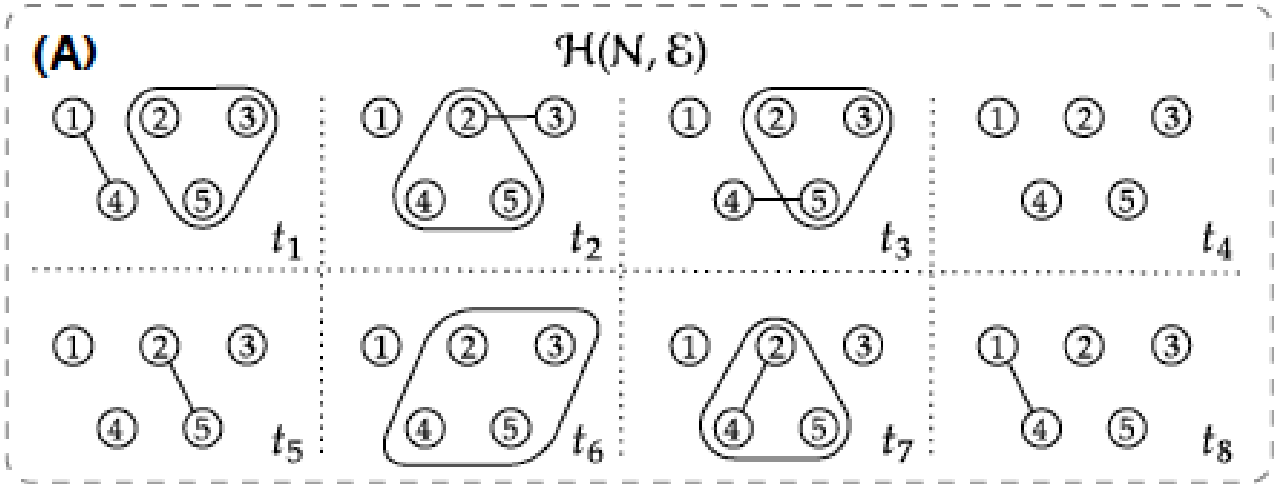
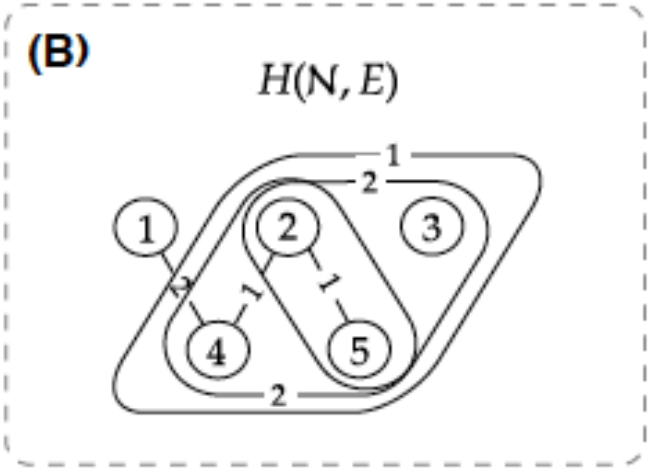


Time-aggregated network
$H(N, E)$



w_j: the number of activations of h_j in $\mathcal{H}(N, \mathcal{E})$

Contagion process on THNs

- **S**usceptible-**I**nfected threshold process:

  - At t_0 a single seed node gets infected;

  - When a hyperlink is active at any time t,

  if the number of infected nodes within the hyperlink $\geq \Theta$,

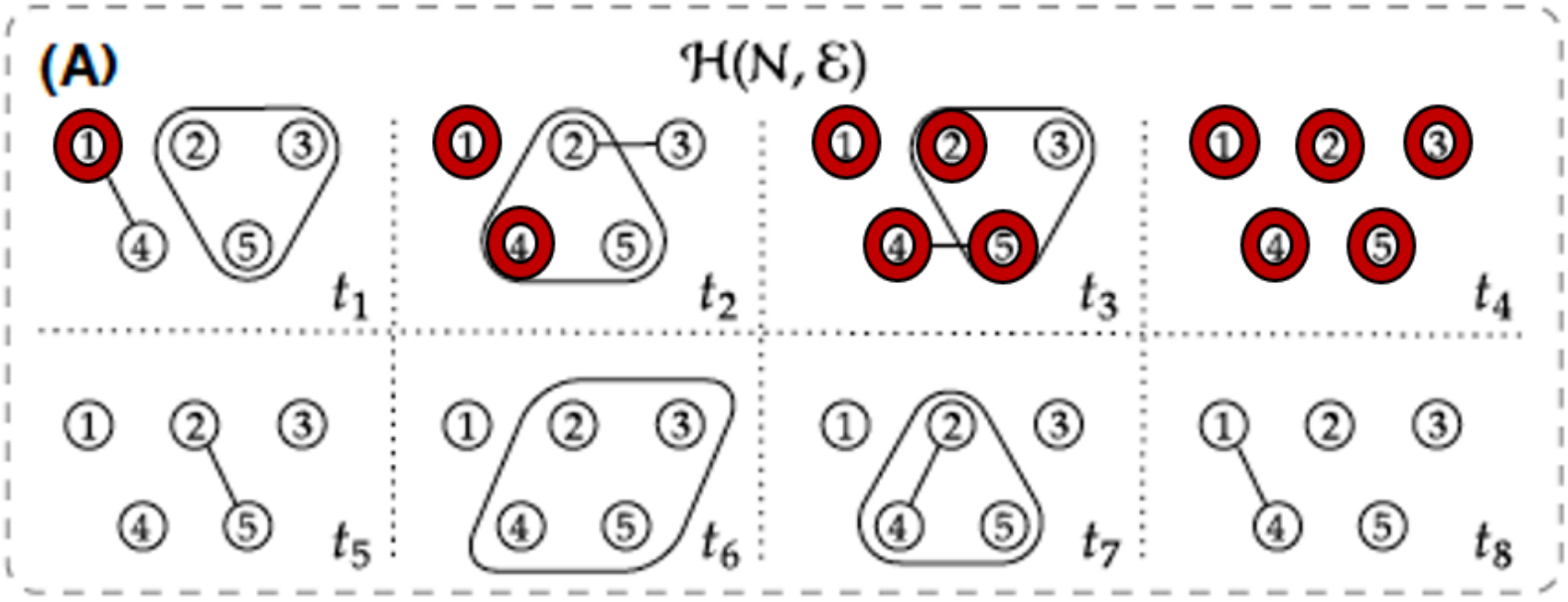
  each susceptible node in the hyperlink gets infected

  independently with probability β at time $t + 1$.

G. de Arruda, G. Petri, and Y. Moreno, Social contagion models on hypergraphs, Phys. Rev. Research **2**, 023032 (2023).

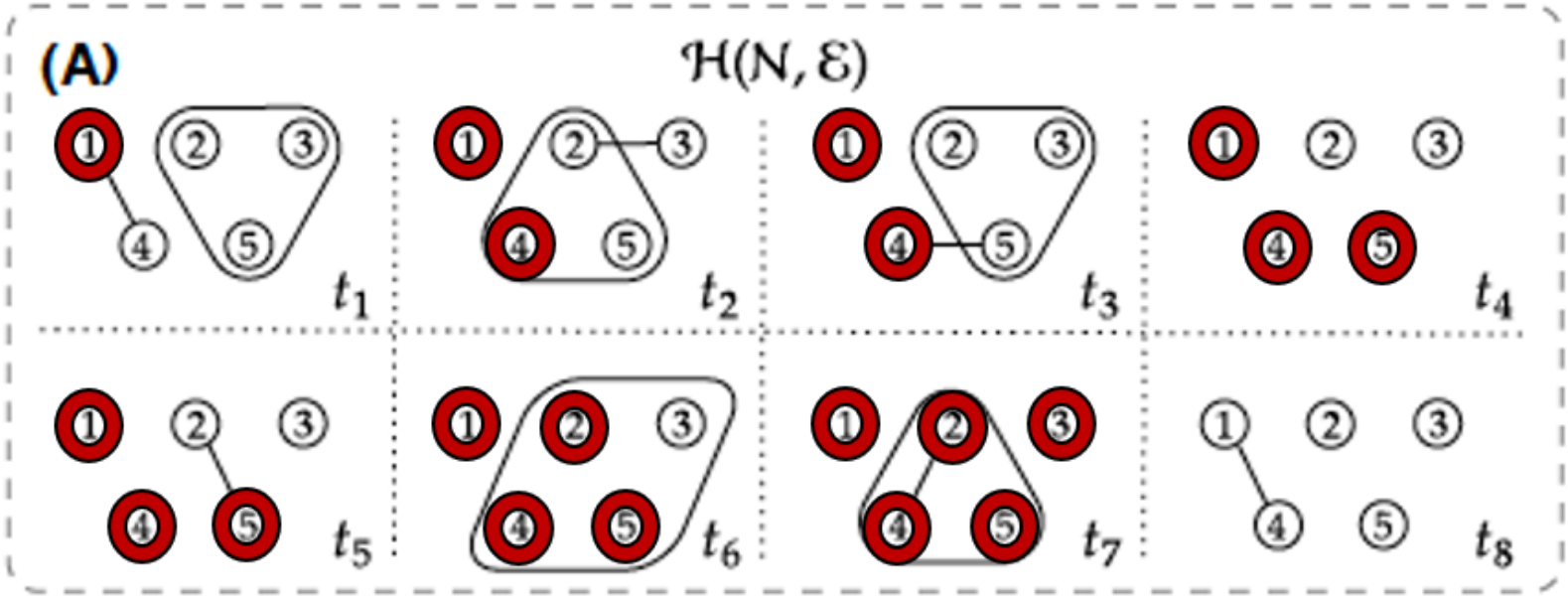
Example – seed: node 1

Threshold $\Theta = 1$, $\beta = 1$



Example – seed: node 1

Threshold $\Theta = d - 1, \beta = 1$



Research question

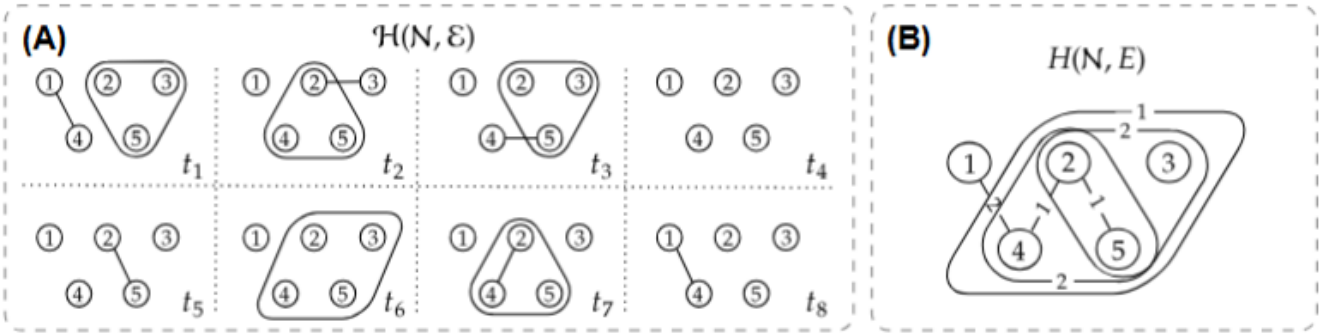
- Which hyperlinks contribute more to the diffusion process?
  - Definition: contribution of each hyperlink

    the number of nodes that are infected directly via the hyperlink

  - Correlation with properties of the hyperlink within the THN

Construction of diffusion backbone

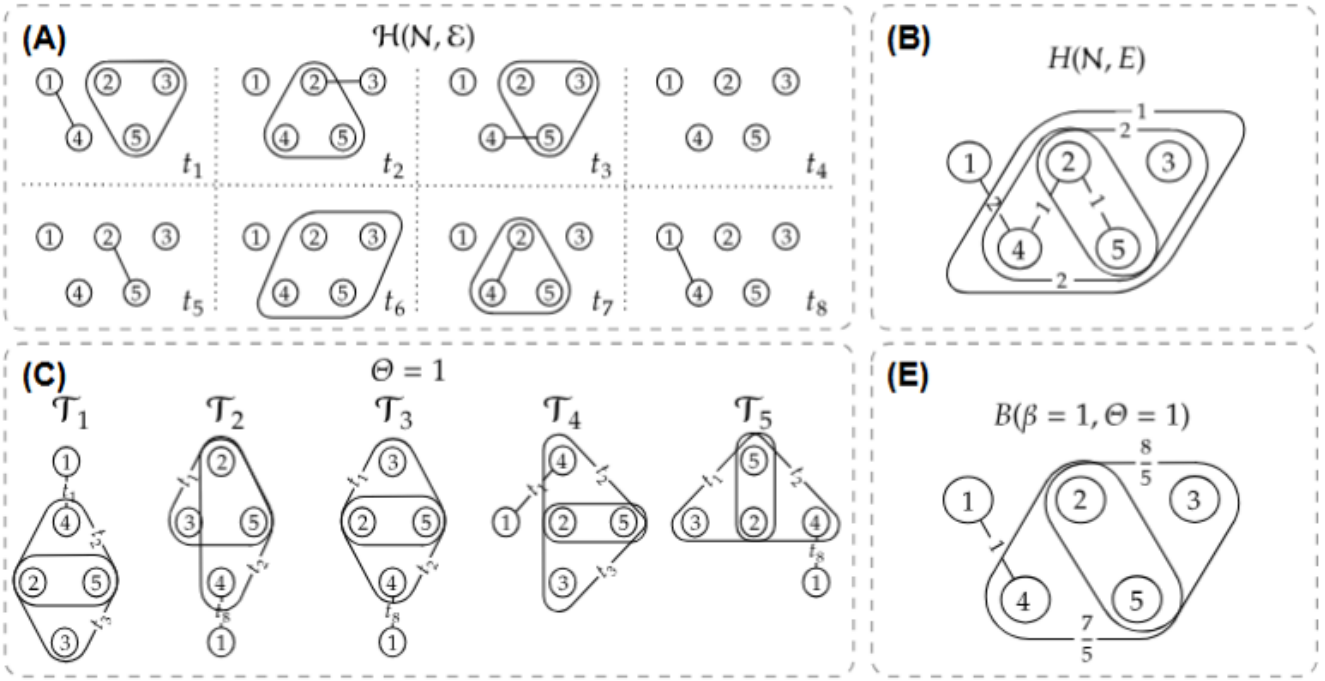
THN
$\mathcal{H}(N, \mathcal{E})$



Construction of diffusion backbone

THN
$\mathcal{H}(N, \mathcal{E})$

$\Theta = 1$
$\beta = 1$



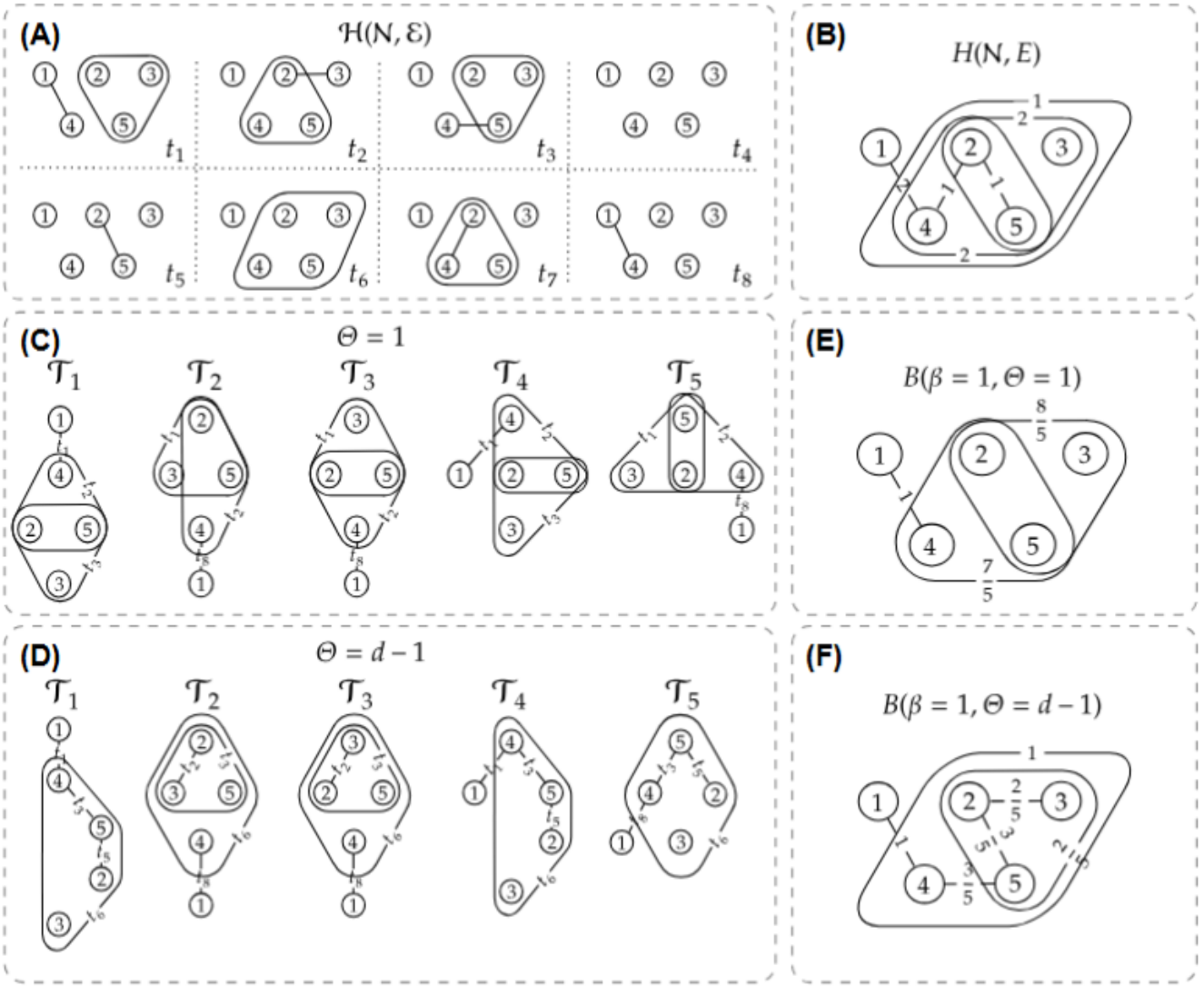
w_j^B: the average number of nodes that are infected directly via hyperlink h_j.

Construction of diffusion backbone

THN
$\mathcal{H}(N, \mathcal{E})$

$\Theta = 1$
$\beta = 1$

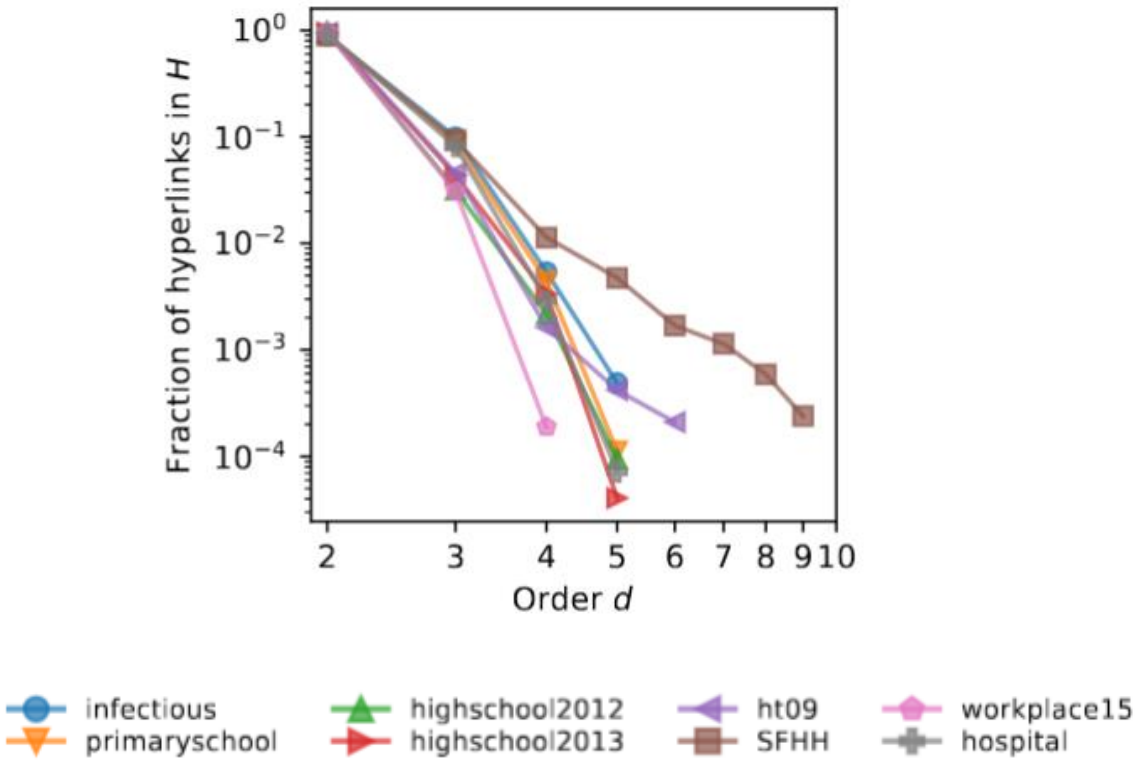
$\Theta = d - 1$
$\beta = 1$



Datasets

- 8 empirical datasets of face-to-face interactions

http://www.sociopatterns.org/datasets/



Backbone $B(\beta \to 0)$

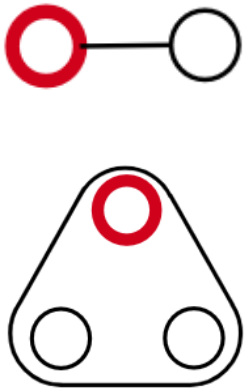
For an arbitrary hyperlink h_j

- $\boldsymbol{\Theta} = 1$

$$w_j^B \approx \beta |h_j| \big(|h_j| - 1\big) w_j / N,$$

- $\boldsymbol{\Theta} = d - 1$

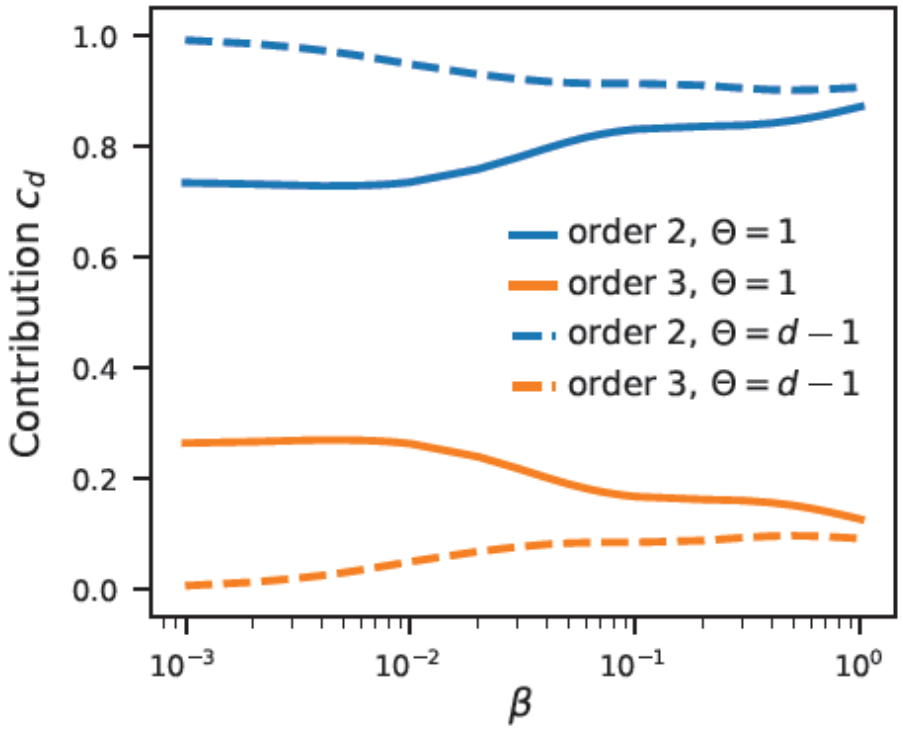
$$w_j^B \approx \begin{cases} \beta \frac{|h_j|(|h_j|-1)w_j}{N}, & |h_j|=2 \\ \mathcal{O}(\beta^2), & |h_j|=3 \end{cases}$$



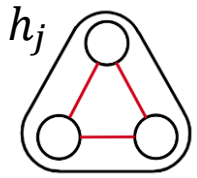
w_j^B is the weight in the backbone B,

w_j is the weight in the time-aggregated network H.

Backbone B as β increases



Centrality metrics for hyperlinks based on local network



$\mathcal{L}^{sub}(j)$: order-2 links that share two nodes with h_j

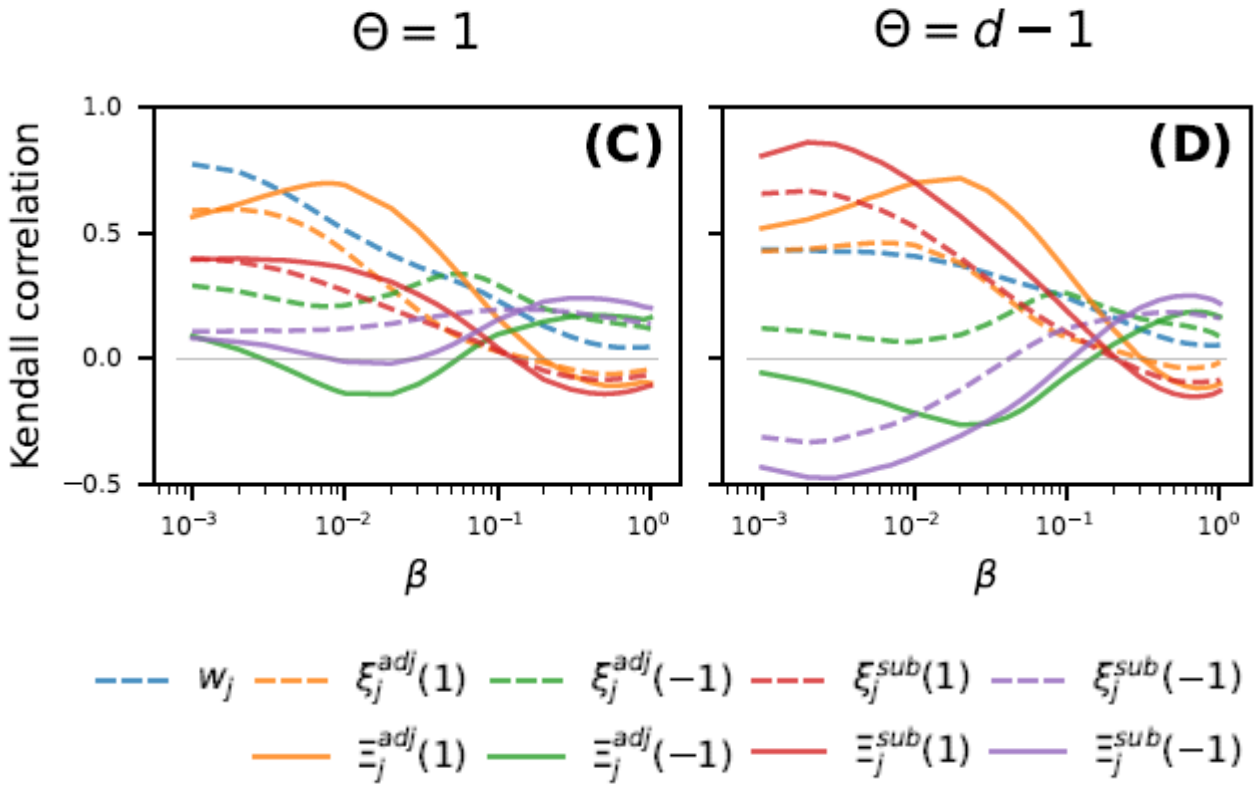
Static $\xi_j^{sub}(\alpha) = w_j \left(1 + \sum_{l \in \mathcal{L}^{sub}(j)} w_l\right)^{\alpha}$,

Temporal $\Xi_j^{sub}(\alpha) = \sum_{t=1}^{T} x_j(t) \left(1 + \sum_{l \in \mathcal{L}^{sub}(j)} \sum_{s<t} x_l(s)\right)^{\alpha}$,

where α is the scaling parameter,

$x_j(t)$ indicates if h_j is active at t, $w_j = \sum_{t=1}^{T} x_j(t)$,

Correlation analysis



Conclusion

- Diffusion backbone characterizes the contribution of each hyperlink,
- The backbone in the limiting case $\beta \to 0$,
- Local centrality metrics for hyperlinks in THNs.

Thanks!