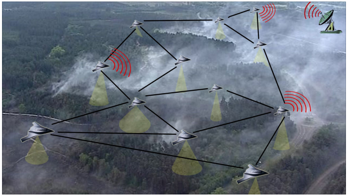
On the Optimality of Sparse Feedback Control Under Architecture-Dependent Communication Delays

Luca Ballotta

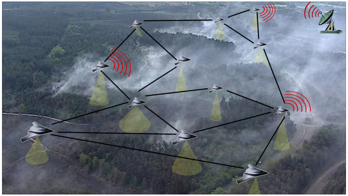
Delft University of Technology

Networked Control Systems are Scaling up

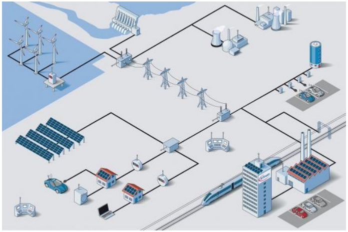


Smart sensor networks

Networked Control Systems are Scaling up

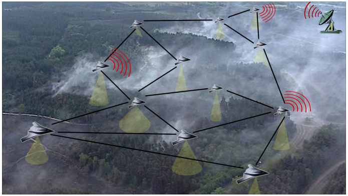


Smart sensor networks

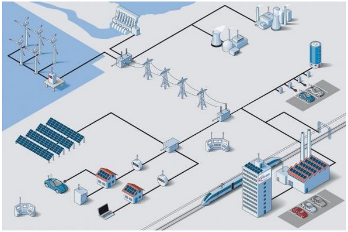


Smart grids

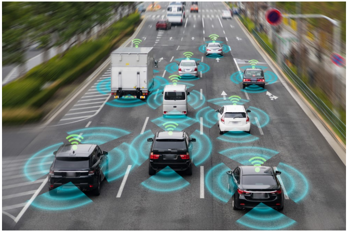
Networked Control Systems are Scaling up



Smart sensor networks

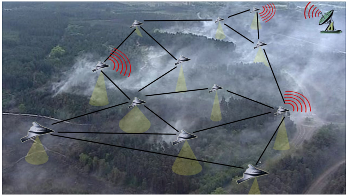


Smart grids

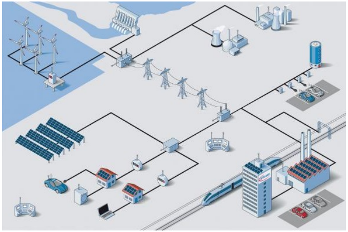


Vehicular networks

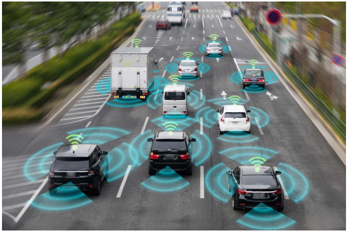
Networked Control Systems are Scaling up



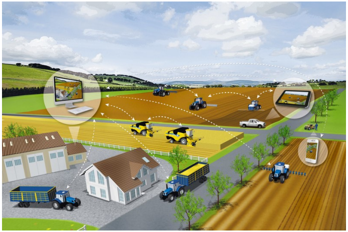
Smart sensor networks



Smart grids

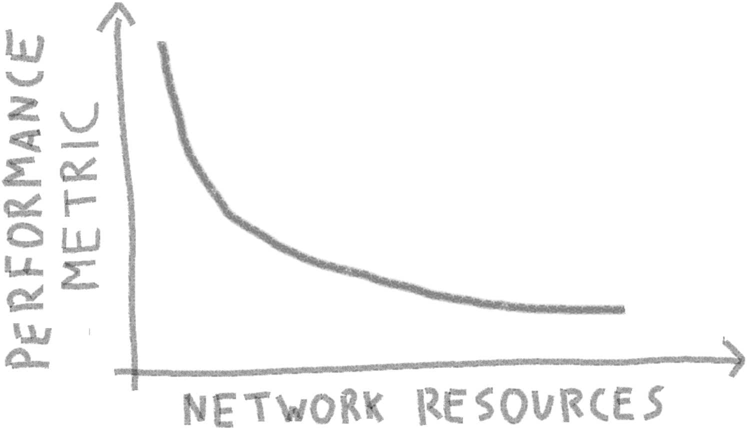


Vehicular networks

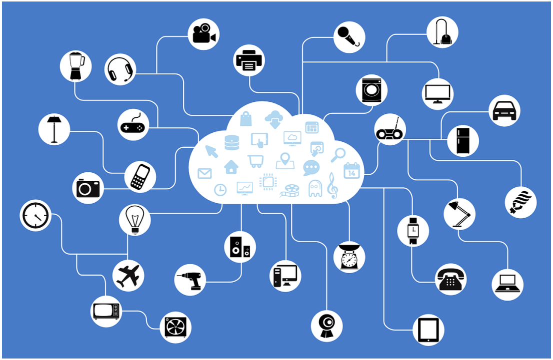


Smart agriculture

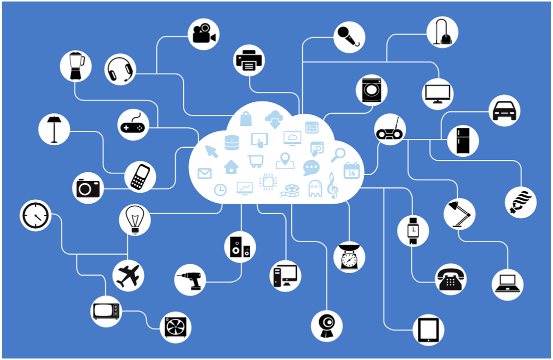
A Classic Tradeoff: Performance vs. Resources



Example: Distributed Control



Example: Distributed Control



Really need to use **all** sensors/links?

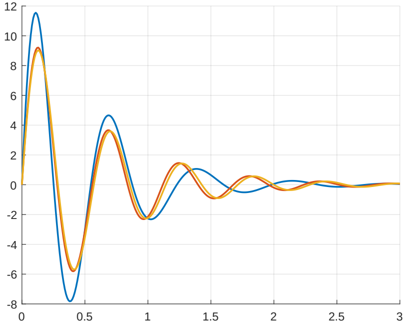
Acknowledgments

Luca Schenato
(Unipd)

Mihailo Jovanović
(USC)

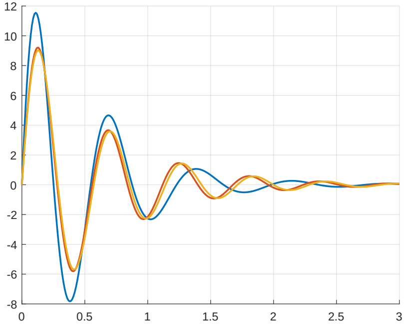
Vijay Gupta
(Purdue)

Controller Architecture: How to Choose?

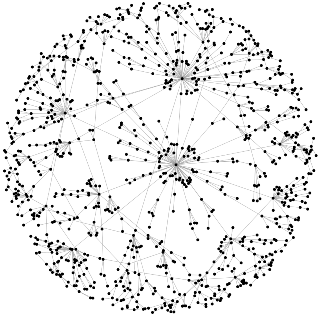


Control performance

Controller Architecture: How to Choose?

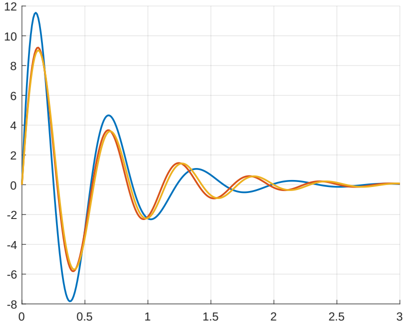


Control performance

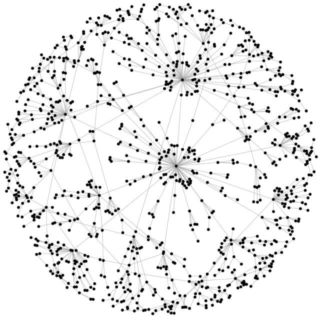


Implementation complexity

Controller Architecture: How to Choose?



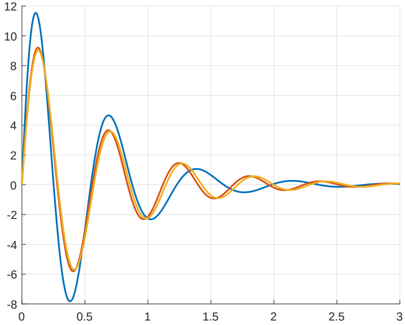
Control performance



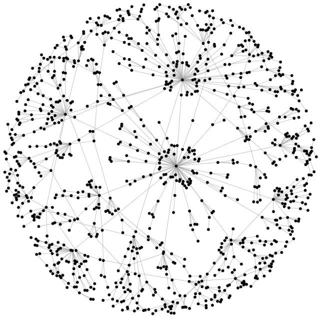
Implementation complexity

➔ **Centralized** controller: **performance**

Controller Architecture: How to Choose?



Control performance

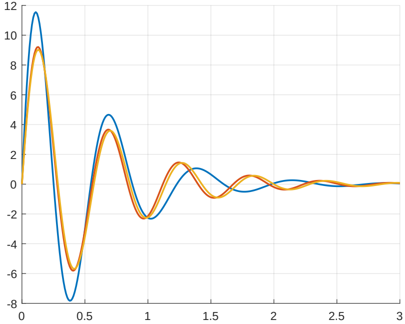


Implementation complexity

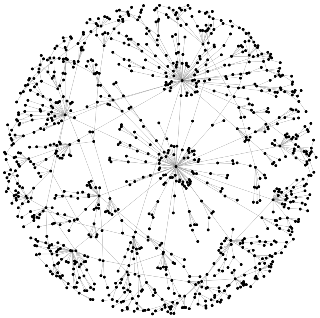
➔ **Centralized** controller: **performance**

➔ **Distributed** controller: **robustness, scalability**...

Controller Architecture: How to Choose?



Control performance



Implementation complexity

➔ **Centralized** controller: **performance(?)**

➔ **Distributed** controller: **robustness, scalability**...

Centralized-Decentralized Trade-off

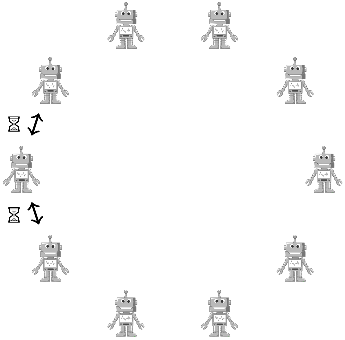
Assumption

Communication **delays** increase with **number of links**

Centralized-Decentralized Trade-off

Assumption

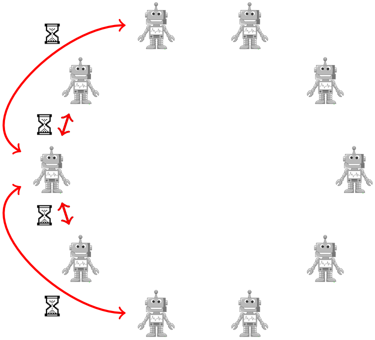
Communication **delays** increase with **number of links**



Centralized-Decentralized Trade-off

Assumption

Communication **delays** increase with **number of links**

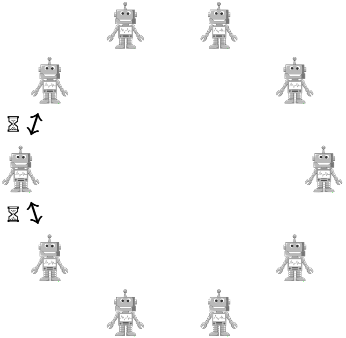


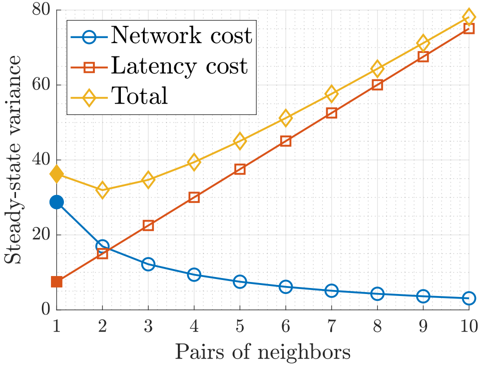
Centralized-Decentralized Trade-off

Assumption

Communication **delays** increase with **number of links**

Optimal architecture is sparse, in general



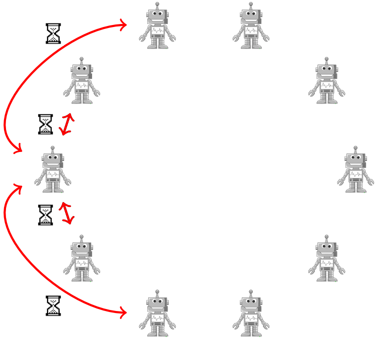


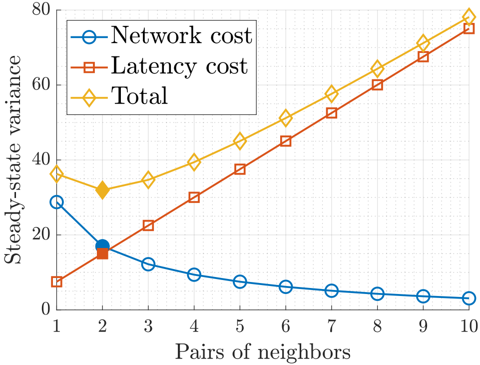
Centralized-Decentralized Trade-off

Assumption

Communication **delays** increase with **number of links**

Optimal architecture is sparse, in general





Setup: Consensus in Undirected Network

Agent i with state (error) x_i

Setup: Consensus in Undirected Network

Agent i with state (error) x_i

➔ feedback from n neighbors

Setup: Consensus in Undirected Network

Agent i with state (error) x_i

- ➔ feedback from n neighbors
- ➔ communication delay $\tau_n \uparrow n$

Setup: Consensus in Undirected Network

Agent i with state (error) x_i

- ➔ feedback from n neighbors
- ➔ communication delay $\tau_n \uparrow n$
- ➔ symmetric feedback gains $k_1, \ldots, k_n$

Setup: Consensus in Undirected Network

Agent i with state (error) x_i

- ➔ feedback from n neighbors
- ➔ communication delay $\tau_n \uparrow n$
- ➔ symmetric feedback gains $k_1, \dots, k_n$

Continuous-time single integrator ➔ $\boxed{\mathrm{d}x(t) = -Kx(t-\tau_n)\mathrm{d}t + \mathrm{d}w(t)}$

Setup: Consensus in Undirected Network

Agent i with state (error) x_i

- ➔ feedback from n neighbors
- ➔ communication delay $\tau_n \uparrow n$
- ➔ symmetric feedback gains $k_1, \ldots, k_n$

Continuous-time single integrator ➔ $$\boxed{\mathrm{d}x(t) = -Kx(t-\tau_n)\mathrm{d}t + \mathrm{d}w(t)}$$

To be designed

Setup: Consensus in Undirected Network

Agent i with state (error) x_i

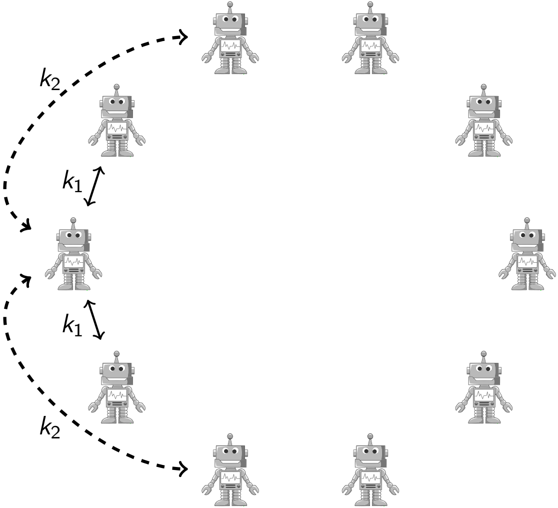
- ➔ feedback from n neighbors
- ➔ communication delay $\tau_n \uparrow n$
- ➔ symmetric feedback gains $k_1, \ldots, k_n$

Continuous-time single integrator ➔ $\boxed{\mathrm{d}x(t) = -Kx(t - \tau_n)\mathrm{d}t + \mathrm{d}w(t)}$

To be designed

Setup: Example with Circulant Topology

$n = 2$ pairs of neighbors



Decoupling the Dynamics

$$\mathrm{d}x(t) = -Kx(t-\tau_n)\mathrm{d}t + \mathrm{d}w(t)$$

Decoupling the Dynamics

$$\mathrm{d}x(t) = -Kx(t-\tau_n)\mathrm{d}t + \mathrm{d}w(t)$$

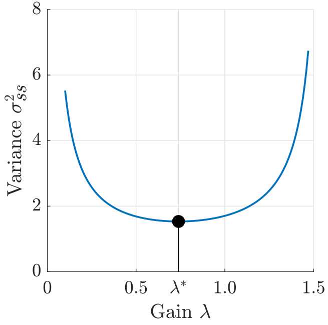
Change of basis ➔ $\boxed{\mathrm{d}\tilde{x}_j(t) = -\lambda_j\tilde{x}_j(t-\tau_n)\mathrm{d}t + \mathrm{d}\tilde{w}_j(t)} \quad j = 1,\ldots,N$

Decoupling the Dynamics

$$\mathrm{d}x(t) = -Kx(t-\tau_n)\mathrm{d}t + \mathrm{d}w(t)$$

Change of basis ➔ $\boxed{\mathrm{d}\tilde{x}_j(t) = -\lambda_j\tilde{x}_j(t-\tau_n)\mathrm{d}t + \mathrm{d}\tilde{w}_j(t)}$ $\quad j = 1, \ldots, N$

Steady-state variance[1]: $\sigma^2_{ss}(\lambda_j) = \dfrac{1+\sin(\lambda_j\tau_n)}{2\lambda_j\cos(\lambda_j\tau_n)}$

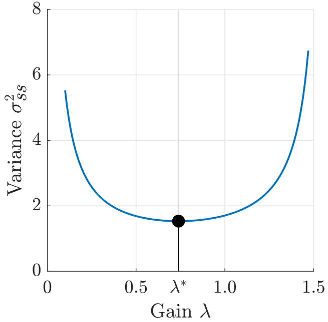


Decoupling the Dynamics

$$\mathrm{d}x(t) = -Kx(t-\tau_n)\mathrm{d}t + \mathrm{d}w(t)$$

Change of basis ➔ $\boxed{\mathrm{d}\tilde{x}_j(t) = -\lambda_j \tilde{x}_j(t-\tau_n)\mathrm{d}t + \mathrm{d}\tilde{w}_j(t)}$ $\quad j = 1, \dots, N$

Steady-state variance[1]: $\sigma^2_{ss}(\lambda_j) = \dfrac{1+\sin(\lambda_j\tau_n)}{2\lambda_j\cos(\lambda_j\tau_n)}$ $\quad \boxed{\lambda_j \in \left(0, \dfrac{\pi}{2\tau_n}\right)}$



Optimal Mean-Square Consensus

Problem

Choose the feedback gains that minimize the steady-state variance:

$$\arg\min_{K} \ \mathbb{E}\left[\lim_{t\to\infty} \|x(t)\|^2\right]$$

Optimal Mean-Square Consensus

Problem

Choose the feedback gains that minimize the steady-state variance:

$$\arg\min_{K} \ \mathbb{E}\left[\lim_{t\to\infty} \|x(t)\|^2\right]$$

Equivalently: $$\arg\min_{K} \sum_{\lambda_j=2}^{N} \sigma_{ss}^2(\lambda_j)$$

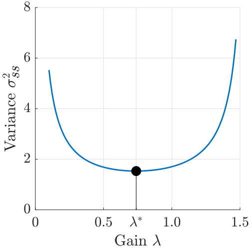
Optimal Mean-Square Consensus

Problem

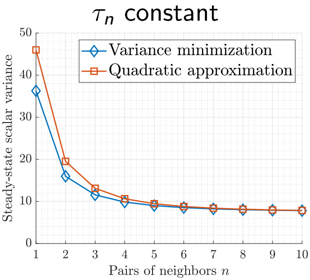
Choose the feedback gains that minimize the steady-state variance:

$$\arg\min_{K} \mathbb{E}\left[\lim_{t\to\infty} \|x(t)\|^2\right]$$

Equivalently: $\arg\min_{K} \sum_{\lambda_j=2}^{N} \sigma_{ss}^2(\lambda_j)$ **convex problem**

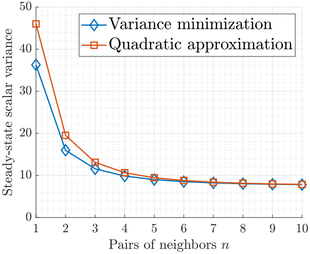


Circular Formation: Decentralized-Centralized Trade-off

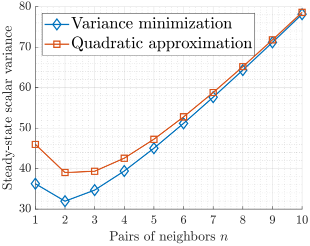


Circular Formation: Decentralized-Centralized Trade-off

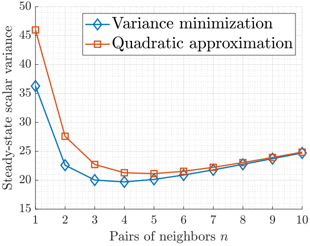
τ_n constant

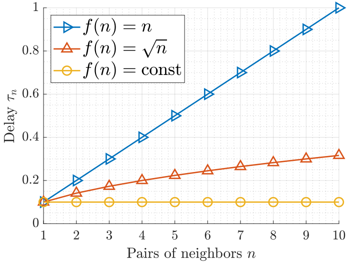


$\tau_n \propto n$



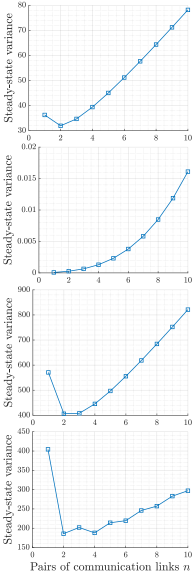
$\tau_n \propto \sqrt{n}$





Extensions to More Realistic Dynamics

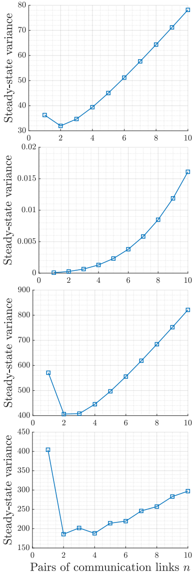
More realistic



$$\mathrm{d}x_i(t) = u_{P,i}(t)\mathrm{d}t + \mathrm{d}w_i(t)$$

Extensions to More Realistic Dynamics

More realistic



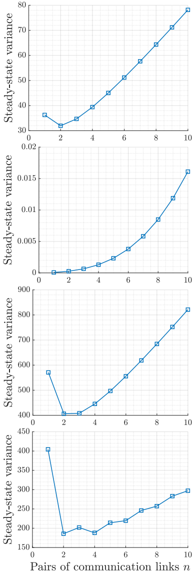
$$\mathrm{d}x_i(t) = u_{P,i}(t)\mathrm{d}t + \mathrm{d}w_i(t)$$

$$\mathrm{d}x_i(t) = z_i(t)\mathrm{d}t$$
$$\mathrm{d}z_i(t) = \eta(-z_i(t) + u_{P,i}(t))\mathrm{d}t + \mathrm{d}w_i(t)$$

Inertia

Extensions to More Realistic Dynamics

More realistic



$$\mathrm{d}x_i(t) = {\color{red}u_{P,i}(t)}\mathrm{d}t + \mathrm{d}w_i(t)$$

Inertia

$$\mathrm{d}x_i(t) = z_i(t)\mathrm{d}t$$
$$\mathrm{d}z_i(t) = {\color{red}\eta}(-z_i(t) + {\color{red}u_{P,i}(t)})\mathrm{d}t + \mathrm{d}w_i(t)$$

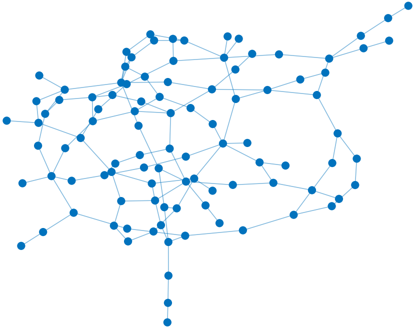
Wireless communication

$$x_i(k+1) = x_i(k) + {\color{red}u_{P,i}(k)} + w_i(k)$$

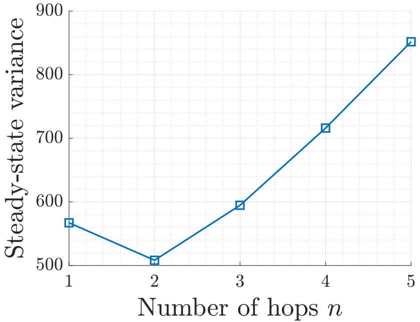
$$x_i(k+1) = x_i(k) + z_i(k)$$
$$z_i(k+1) = z_i(k) + {\color{red}\eta}(-z_i(k) + {\color{red}u_{P,i}(k)}) + w_i(k)$$

Extensions to Undirected Network

Network topology



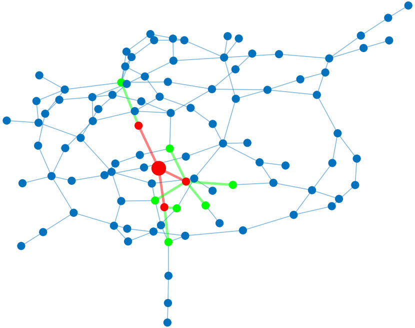
$\tau_n \propto n$



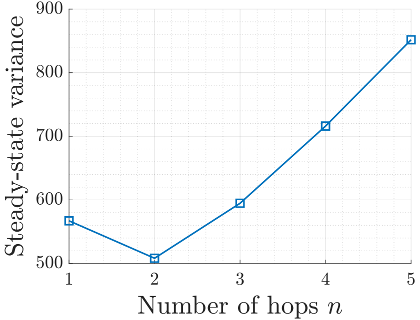
L. Ballotta, M. R. Jovanović, L. Schenato, "Can Decentralized Control Outperform Centralized? The Role of Communication Latency," IEEE TCNS 2023

Extensions to Undirected Network

Network topology



$\tau_n \propto n$



L. Ballotta, M. R. Jovanović, L. Schenato, "Can Decentralized Control Outperform Centralized? The Role of Communication Latency," IEEE TCNS 2023

Deterministic Protocol: Maximize Convergence Speed

Discrete-time single integrator ➔ $\boxed{x(k+1) = x(k) - Kx(k-\tau_n)}$

Deterministic Protocol: Maximize Convergence Speed

Discrete-time single integrator ➔ $\boxed{x(k+1) = x(k) - K x(k - \tau_n)}$

Characteristic polynomials ➔ $h(z, \lambda) = z^{\tau_n + 1} - z^{\tau_n} + \lambda, \quad \lambda_1, \ldots, \lambda_N = \mathrm{eig}(K)$

➔ $(\tau_n + 1)N$ modes

Deterministic Protocol: Maximize Convergence Speed

Discrete-time single integrator ➔ $\boxed{x(k+1) = x(k) - Kx(k-\tau_n)}$

Characteristic polynomials ➔ $h(z,\lambda) = z^{\tau_n+1} - z^{\tau_n} + \lambda, \quad \lambda_1, \ldots, \lambda_N = \mathrm{eig}(K)$

➔ $(\tau_n + 1)N$ modes

➔ Convergence speed determined by the slowest mode(s) < 1

Deterministic Protocol: Maximize Convergence Speed

Discrete-time single integrator ➔ $x(k+1) = x(k) - Kx(k - \tau_n)$

Characteristic polynomials ➔ $h(z, \lambda) = z^{\tau_n + 1} - z^{\tau_n} + \lambda, \quad \lambda_1, \ldots, \lambda_N = \mathrm{eig}(K)$

➔ $(\tau_n + 1)N$ modes

➔ Convergence speed determined by the slowest mode(s) < 1

Problem

Choose the feedback gains that minimize the convergence rate:

Deterministic Protocol: Maximize Convergence Speed

Discrete-time single integrator ➔ $\boxed{x(k+1) = x(k) - Kx(k-\tau_n)}$

Characteristic polynomials ➔ $h(z,\lambda) = z^{\tau_n+1} - z^{\tau_n} + \lambda, \quad \lambda_1, \dots, \lambda_N = \mathrm{eig}(K)$

➔ $(\tau_n + 1)N$ modes

➔ Convergence speed determined by the slowest mode(s) < 1

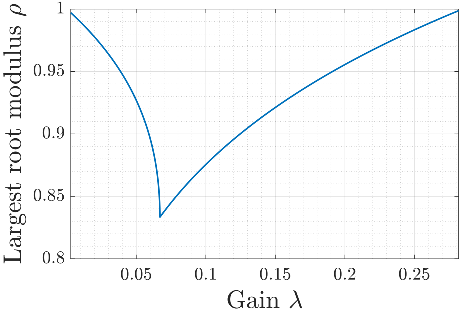
Problem

Choose the feedback gains that minimize the convergence rate:

$$K^* = \arg\min_K \underbrace{\max_{z,\lambda\neq\lambda_1} \{|z| : h(z,\lambda) = 0\}}_{\textit{slowest modes}}$$

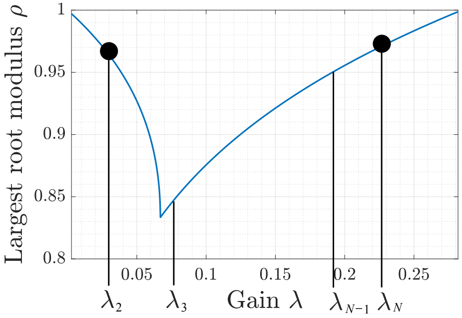
Sneak Peek Into Solution Approach

Slowest mode $\max_z\{|z| : h(z,\lambda) = 0\}$ vs. eigenvalue λ:



Sneak Peek Into Solution Approach

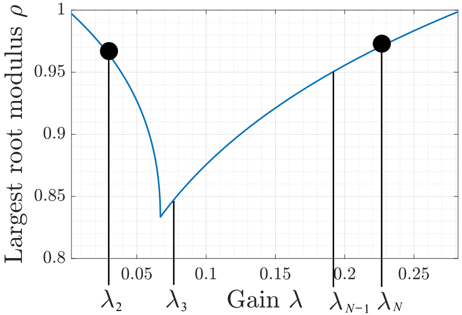
Slowest mode $\max_z\{|z| : h(z,\lambda) = 0\}$ vs. eigenvalue λ:



➔ The two slowest modes correspond to λ_2 and λ_N

Sneak Peek Into Solution Approach

Slowest mode $\max_z\{|z| : h(z,\lambda) = 0\}$ vs. eigenvalue λ:

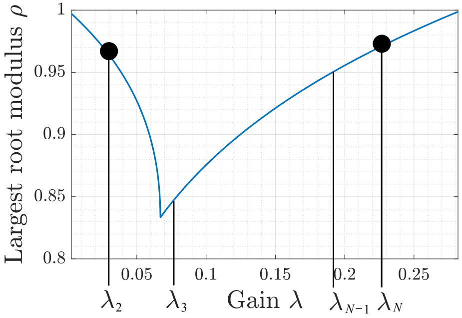


➔ The two slowest modes correspond to λ_2 and λ_N

➔ They must be equal!

Sneak Peek Into Solution Approach

Slowest mode $\max_z\{|z| : h(z,\lambda) = 0\}$ vs. eigenvalue λ:



➔ The two slowest modes correspond to λ_2 and λ_N

➔ They must be equal!

➔ Can be **exactly** solved through an SDP + an algebraic equation

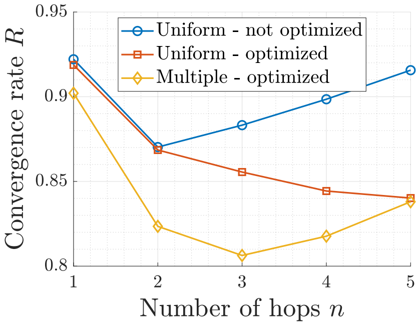
Convergence Speed: Decentralized-Centralized Trade-off

Uniform gains: $K = -gL$, L Laplacian matrix

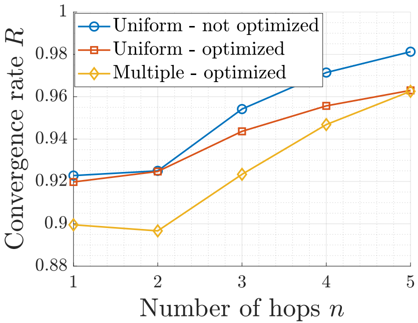
Convergence Speed: Decentralized-Centralized Trade-off

Uniform gains: $K = -gL$, L Laplacian matrix

$\tau_n \propto n$



$\tau_n \propto n^2$



L. Ballotta, V. Gupta, "Faster Consensus via a Sparser Controller," IEEE L-CSS 2023

Take-Home Message and Future Directions

Communication delays determine the optimal controller:

Take-Home Message and Future Directions

Communication delays determine the optimal controller:

- optimal architecture depends on how delays change with number of links

Take-Home Message and Future Directions

Communication delays determine the optimal controller:

- optimal architecture depends on how delays change with number of links
- stochastic dynamics appear to require sparser control architectures

Take-Home Message and Future Directions

Communication delays determine the optimal controller:

- optimal architecture depends on how delays change with number of links
- stochastic dynamics appear to require sparser control architectures

To the moon:

- Other control settings (robust, nonlinear...)

Take-Home Message and Future Directions

Communication delays determine the optimal controller:

- optimal architecture depends on how delays change with number of links
- stochastic dynamics appear to require sparser control architectures

To the moon:

- Other control settings (robust, nonlinear...)
- Broader communication constraints (SLS, TV networks/delays...)

Take-Home Message and Future Directions

Communication delays determine the optimal controller:

- optimal architecture depends on how delays change with number of links
- stochastic dynamics appear to require sparser control architectures

To the moon:

- Other control settings (robust, nonlinear...)
- Broader communication constraints (SLS, TV networks/delays...)

Thank you for your attention!

l.ballotta@tudelft.nl