Hodge-Aware Subspace Detector

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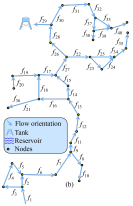
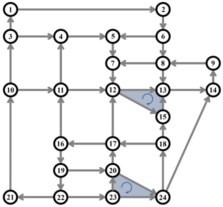
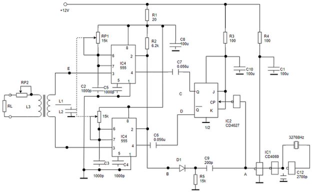
Motivation



Water flow



Traffic flow

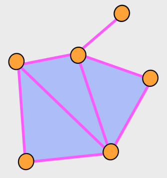
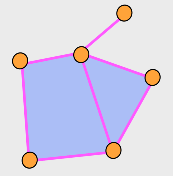
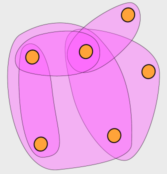


Electric current

- Water networks: detecting water leakage
- Traffic networks: detecting traffic jams
- Power networks: detecting outages

Motivation

- Detecting anomalies in signals with multi-way dependencies

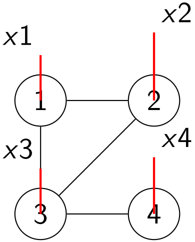




Challenge

- Data contain an irregular structure
- A detector with theoretical guarantees

Graph Structure of Node Signals

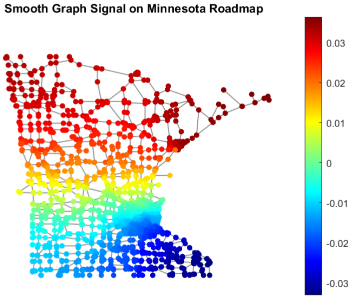
- Graph $\mathcal{G}$ with Laplacian matrix $\mathbf{L}$
- Graph signals are mappings $x : \mathcal{V} \to \mathbb{R}$

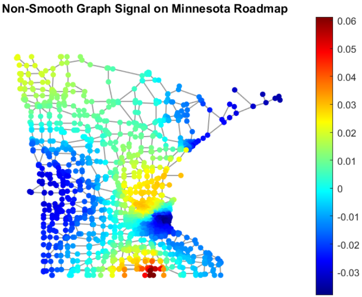


$$\mathbf{L} = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \qquad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

Detection with Graph Signals

- Normal behavior – smooth signal $\mathbf{x}^\top \mathbf{L} \mathbf{x}$ is small
- Anomaly – nonsmooth $\mathbf{x}^\top \mathbf{L} \mathbf{x}$ is high





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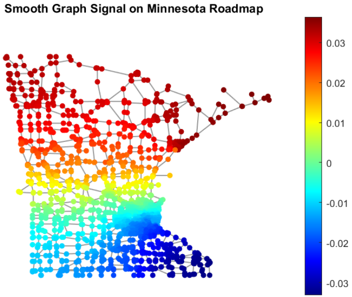
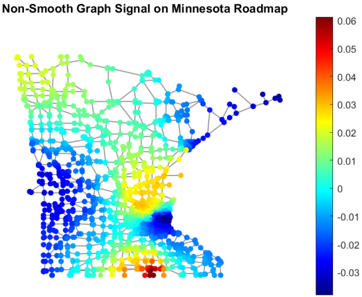


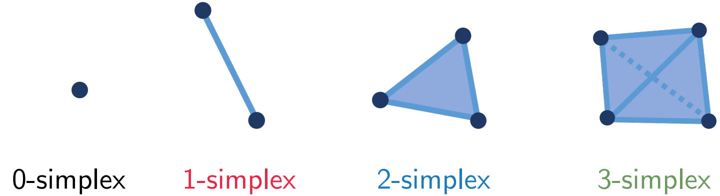


Goal: Develop a detection theory for signals on edges and beyond

Topological Data Structure

Simplicial complexes

- 0-simplices: nodes – single element
- 1-simplices: edges – pair-wise relationships
- 2-simplices: triangles – relationships of 3 elements
- 3-simplices: tetrahedrons relationships of 4 elements
- ...



Simplicial Signals

- Mapping[1] from simplices to $\mathbb{R}$
- We collect $k-$signal into the vector $\mathbf{s}^k = \left[s_1^k, \ldots, s_{N_k}^k\right]^\top$

[1] S. Barbarossa and S. Sardellitti (2020). "Topological signal processing over simplicial complexes". In: *IEEE Transactions on Signal Processing* 68, pp. 2992–3007

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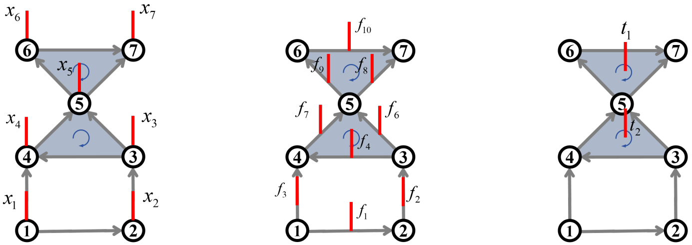
- Node signals: Measurements of temperature, water pressure
- Edge signals: Measurement of water flow, traffic flow
- Triangle signals: EEG measurements of three voxel interaction



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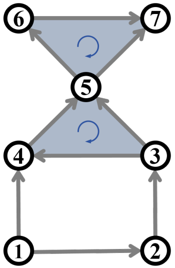
Algebraic Representation of Simplicial Complexes

Topology can be represented by incidence matrices

- $\mathbf{B}_1$: node-to-edge incidence matrix
- $\mathbf{B}_2$: edge-to-triangle incidence matrix
- ...

$$\mathbf{B}_1 = \left.\overbrace{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}}^{\textit{edges}}\right\}\textit{nodes} \quad \mathbf{B}_2 = \left.\overbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}}^{\textit{triangles}}\right\}\textit{edges}$$



Hodge Laplacians

The whole structure is represented by the Hodge Laplacians:

$$
\begin{aligned}
\mathbf{L}_0 &= \mathbf{B}_1\mathbf{B}_1^\top \\
\mathbf{L}_k &= \mathbf{B}_k^\top\mathbf{B}_k + \mathbf{B}_{k+1}\mathbf{B}_{k+1}^\top \quad k = 1, \ldots, K-1 \\
\mathbf{L}_K &= \mathbf{B}_K^\top\mathbf{B}_K
\end{aligned}
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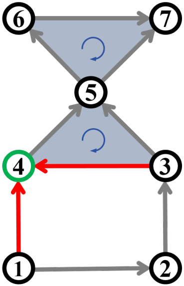
Two terms in Hodge Laplacians $\mathbf{L}_k$:

- Lower Laplacian: $\mathbf{L}_{k,\ell} = \mathbf{B}_k^\top\mathbf{B}_k$ represents lower adjacencies
- Upper Laplacian: $\mathbf{L}_{k,u} = \mathbf{B}_{k+1}\mathbf{B}_{k+1}^\top$ represents upper adjacencies

Hodge Laplacians

- Lower Laplacian: $\mathbf{L}_{k,\ell} = \mathbf{B}_k^\top \mathbf{B}_k$ represents lower adjacencies
- Edges (3, 4) and (1, 4) are lower adjacent



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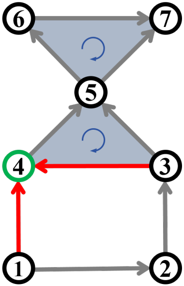

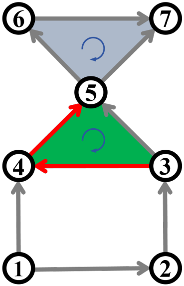

- Upper Laplacian: $\mathbf{L}_{k,u} = \mathbf{B}_{k+1} \mathbf{B}_{k+1}^\top$ represents upper adjacencies
- Edge (3, 4) and (4, 5) are upper adjacent

Hodge Decomposition

- Hodge Laplacian structure

$$\mathbf{L}_k = \mathbf{B}_k^\top \mathbf{B}_k + \mathbf{B}_{k+1} \mathbf{B}_{k+1}^\top$$

Hodge Decomposition

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$$\mathbf{L}_k = \mathbf{B}_k^\top \mathbf{B}_k + \mathbf{B}_{k+1} \mathbf{B}_{k+1}^\top$$

- The space of $k-$signal can be decomposed into three subspaces

$$\mathbb{R}^{N_k} \equiv \mathrm{im}\left(\mathbf{B}_k^\top\right) \oplus \ker\left(\mathbf{L}_k\right) \oplus \mathrm{im}\left(\mathbf{B}_{k+1}\right)$$

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- Any edge flow signal can be decomposed as

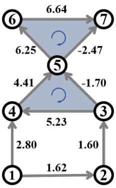
$$\mathbf{f} = \mathbf{f}_{\mathrm{G}} + \mathbf{f}_{\mathrm{C}} + \mathbf{f}_{\mathrm{H}}$$

Hodge Components

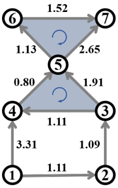
Edge flow signal can be decomposed as

$$\mathbf{f} = \mathbf{f}_{\mathrm{G}} + \mathbf{f}_{\mathrm{C}} + \mathbf{f}_{\mathrm{H}}$$

- Gradient flow $\mathbf{f}_{\mathrm{G}}$: Difference of node signals



Edge flow $\mathbf{f}$



$\mathbf{f}_{\mathrm{G}}$

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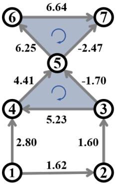
- Gradient flow $\mathbf{f}_{\mathrm{G}}$: Difference of node signals
- Curl flow $\mathbf{f}_{\mathrm{C}}$: Local flows circulating around triangles



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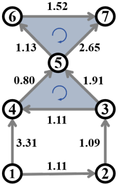

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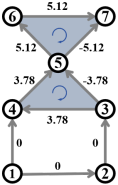

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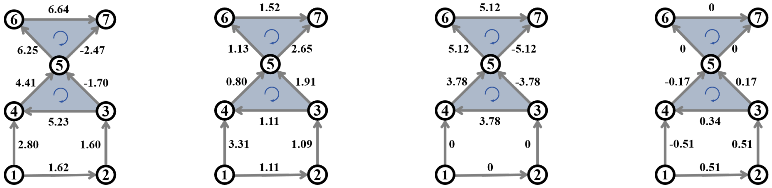
- Gradient flow $\mathbf{f}_{\mathrm{G}}$: Difference of node signals
- Curl flow $\mathbf{f}_{\mathrm{C}}$: Local flows circulating around triangles
- Harmonic flow $\mathbf{f}_{\mathrm{H}}$: Global circulating flow



Edge flow $\mathbf{f}$ $\mathbf{f}_{\mathrm{G}}$ $\mathbf{f}_{\mathrm{C}}$ $\mathbf{f}_{\mathrm{H}}$

Hodge Decomposition

- Eigendecomposition of the first Hodge Laplacian $\mathbf{L}_1 = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^\top$
- The eigenvectors $\mathbf{U}$ can be rearranged into $[\mathbf{U}_{\mathrm{G}}\ \mathbf{U}_{\mathrm{C}}\ \mathbf{U}_{\mathrm{H}}]$

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Edge flow signal can be decomposed as

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Hodge eigenvectors

- $\mathbf{U}_\mathrm{G}$ spans the gradient subspace, $\mathbf{f}_\mathrm{G} = \mathbf{U}_\mathrm{G}\hat{\mathbf{f}}_\mathrm{G}$

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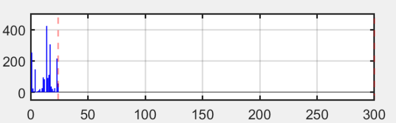
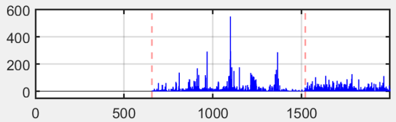
- $\mathbf{U}_\mathrm{C}$ spans the curl subspace, $\mathbf{f}_\mathrm{C} = \mathbf{U}_\mathrm{C}\hat{\mathbf{f}}_\mathrm{C}$
- $\mathbf{U}_\mathrm{H}$ spans the harmonic subspace, $\mathbf{f}_\mathrm{H} = \mathbf{U}_\mathrm{H}\hat{\mathbf{f}}_\mathrm{H}$

Examples of Real-world Flows





Forex

Lastfm

Observation-1: Normally, signal is localized in specific subspace

- Forex flow only has non-curl component
- Lastfm flow only has non-gradient component

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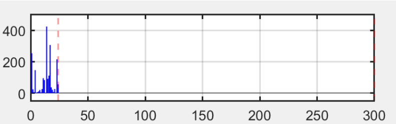

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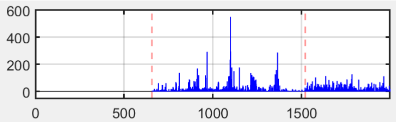

Lastfm

Observation-1: Normally, signal is localized in specific subspace

- Forex flow only has non-curl component
- Lastfm flow only has non-gradient component

Observation-2: Under anomalies, we will have signal subspace spillage

Hypothesis Testing

- $\mathbf{x}$ – true edge flow
- $\mathbf{f} = \mathbf{x} + \mathbf{n}$ –observed edge flow with $\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \sigma^2 \mathbf{I}\right)$.

$\mathcal{H}_0$: $\mathbf{x}$ lives in some specific Hodge subspace
$\mathcal{H}_1$: $\mathbf{x}$ does not live in this Hodge subspace

Hypothesis Testing

Hypothesis $\mathcal{H}_0$: the subspace is spanned by eigenvectors

$$\mathbf{U}_{\Delta} \in \{\mathbf{U}_{\mathrm{G}}, \mathbf{U}_{\mathrm{C}}, \mathbf{U}_{\mathrm{H}}, [\mathbf{U}_{\mathrm{G}}, \mathbf{U}_{\mathrm{C}}], [\mathbf{U}_{\mathrm{G}}, \mathbf{U}_{\mathrm{H}}], [\mathbf{U}_{\mathrm{C}}, \mathbf{U}_{\mathrm{H}}]\}.$$

The complement eigenvectors are $\mathbf{U}_{\overline{\Delta}}$ so that $\mathbf{U}_{\overline{\Delta}}^{\top}\mathbf{U}_{\Delta} = \mathbf{0}$.

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Hypothesis $\mathcal{H}_1$: the projection of $\mathbf{f}$ in the complement subspace is

$$\hat{\mathbf{f}}_{\overline{\Delta}} = \mathbf{U}_{\overline{\Delta}}^{\top}\mathbf{x} + \mathbf{U}_{\overline{\Delta}}^{\top}\mathbf{n} = \hat{\mathbf{x}}_{\overline{\Delta}} + \hat{\mathbf{n}}_{\overline{\Delta}}.$$

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$$\begin{aligned} \mathcal{H}_0 &: \quad \hat{\mathbf{f}}_{\overline{\Delta}} = \hat{\mathbf{n}}_{\overline{\Delta}} \\ \mathcal{H}_1 &: \quad \hat{\mathbf{f}}_{\overline{\Delta}} = \mathbf{U}_{\overline{\Delta}}^\top \mathbf{x} + \hat{\mathbf{n}}_{\overline{\Delta}} \end{aligned} .$$

Simplicial Hodge Detector

Generalized likelihood ratio test (GLRT)

$$T(\hat{\mathbf{f}}_{\overline{\Delta}}) = \frac{p\left(\hat{\mathbf{f}}_{\overline{\Delta}}; \hat{\mathbf{x}}^*_{\overline{\Delta}1}, \mathcal{H}_1\right)}{p\left(\hat{\mathbf{f}}_{\overline{\Delta}}; \hat{\mathbf{x}}^*_{\overline{\Delta}0}, \mathcal{H}_0\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma$$

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Simplicial Hodge Detector (SHD)

Hypothesis $\mathcal{H}_0$: $\hat{\mathbf{x}}^*_{\overline{\Delta}0} = \mathbf{0}$

Hypothesis $\mathcal{H}_1$: $\hat{\mathbf{x}}^*_{\overline{\Delta}1} = \hat{\mathbf{f}}_{\overline{\Delta}}$

The SHD is

$$T_{\mathrm{SHD}}(\hat{\mathbf{f}}_{\overline{\Delta}}) = \|\hat{\mathbf{f}}_{\overline{\Delta}}\|_2^2/\sigma^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma$$

Spectral priors (sparse, low-pass) can be used to estimate the signal

Performance Guarantees

The detector $T_{\mathrm{SHD}}(\hat{\mathbf{f}}_{\overline{\Delta}})$ has a Chi-square distribution

$$T_{\mathsf{SHD}}(\hat{\mathbf{f}}_{\overline{\Delta}}) \sim \begin{cases} \chi^2_{N_{\overline{\Delta}}} & \text{under } \mathcal{H}_0 \\ \chi^2_{N_{\overline{\Delta}}}(\delta) & \text{under } \mathcal{H}_1 \end{cases}$$

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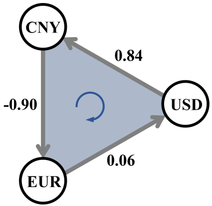
The probability of false alarm and detection are

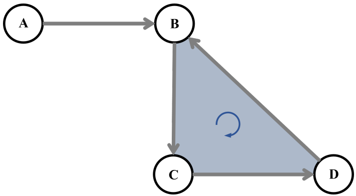
$$\mathsf{P}_{\mathrm{FA}} \triangleq \mathsf{Pr}\{T_{\mathrm{SHD}}(\hat{\mathbf{f}}_{\overline{\Delta}}) > \gamma; \mathcal{H}_0\} = Q_{\chi^2_{N_{\overline{\Delta}}}}(\gamma)$$

$$\mathsf{P}_{\mathsf{D}} \triangleq \mathsf{Pr}\{T_{\mathrm{SHD}}(\hat{\mathbf{f}}_{\overline{\Delta}}) > \gamma; \mathcal{H}_1\} = Q_{\chi^2_{N_{\Delta}}(\delta)}(\gamma)$$

Experimental Setup

Dataset	$\mathcal{H}_0$	$\mathcal{H}_1$
Forex	Gradient flow	$\mathbf{x} = \mathbf{B}_2\mathbf{t},\ \mathbf{t} \sim \mathcal{N}\left(\mathbf{0}, 0.3 \times 10^{-1}\mathbf{I}\right)$
Lastfm	Non-gradient flow	$\mathbf{x} = \mathbf{B}_1^\top\mathbf{v},\ \mathbf{v} \sim \mathcal{N}\left(\mathbf{0}, 5.0 \times 10^{1}\mathbf{I}\right)$
Chicago	$\mathbf{x} = \mathbf{B}_2\mathbf{t},\ \mathbf{t} \sim \mathcal{N}\left(\mathbf{0}, 2.3 \times 10^{-2}\mathbf{I}\right)$	$\hat{\mathbf{x}}_{G,i} \sim \mathcal{N}\left(\exp(-i/5)^\top, 0.01\right)$





Results

SNR = -12 dB

Dataset	SHD-Th.	SHD-Exp.	B-SMSD
Forex	0.71	0.71	0.45
Lastfm	0.99	0.99	0.39
Chicago	0.75	0.75	0.52

- Theoretical and empirical results match each other
- Line-graph detector B-SMSD doesn't consider high-order structures

Conclusion

- A matched subspace detection theory for topological signals
- Relied on Hodge subspace signal projections
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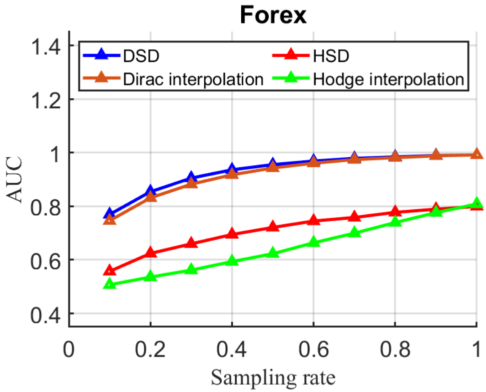
Thank you!

Extended Results-Matched Topological Subspace Detector

Detector with missing values

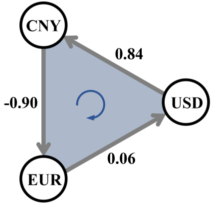
$$T(\mathbf{f}) = \frac{\|\mathbf{f} - \mathbf{U}_{\Delta\Theta}\hat{\boldsymbol{x}}_0^*\|_2^2 - \|\mathbf{f} - \mathbf{U}_{\Theta}\hat{\boldsymbol{x}}_1^*\|_2^2}{\sigma^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma$$



Datasets

Forex(real-world)

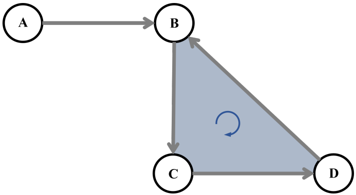
- Exchange rates between 25 different currencies
- 25 nodes, 300 edges and 2300 triangles
- Nodes: 25 different currencies
- Edges: logarithm exchange rate
- Triangles: relationships for three different currencies
- Task: Detecting whether the currency exchange flow is curl-free



Datasets

Lastfm(real-world)

- Users switching artists while playing music
- 657 nodes, 1997 edges and 1276 triangles
- Nodes: 657 different artists
- Edges: transitions between two artists
- Triangles: relationships for three different artists
- Task: Detecting whether the switch flow is divergence-free



Datasets

Chicago(synthetic)

- Traffic network of Chicago
- 546 nodes, 1088 edges and 112 triangles
- Nodes: 546 different junctions
- Edges: 1088 roads between junctions
- Triangles: areas enclosed by three roads
- Task: Detecting whether the synthetic flow is divergence-free